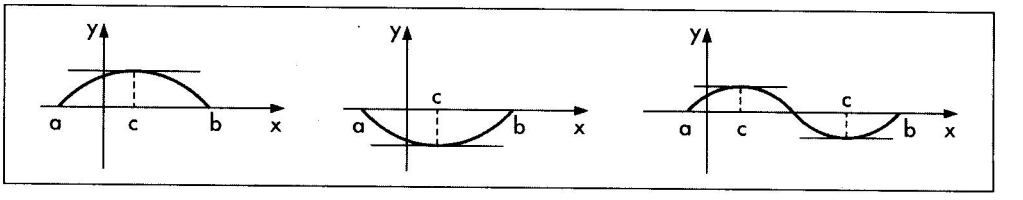
Demonstração

Se $f(x) = 0$, para todo $x \in]a, b[$, basta escolhermos c como qualquer valor do intervalo. Consideremos as situações da Figura B.2.

Figura B.2: Ilustração do Teorema de Rolle.



Se o valor máximo de f for positivo, escolhemos c como ponto de máximo de f. Como $f(a) = f(b) = 0$, temos que $a < c < b$. Então $f'(c) = 0$ pelo teorema B.2.

Se o valor mínimo de f for negativo, escolhemos c como ponto de mínimo de f. Novamente $a < c < b$ e $f'(c) = 0$ pelo teorema B.2.

Podemos ter os dois casos anteriores simultaneamente, como mostra o último gráfico da Figura B.2.

Teorema B.4 (Teorema do valor médio)

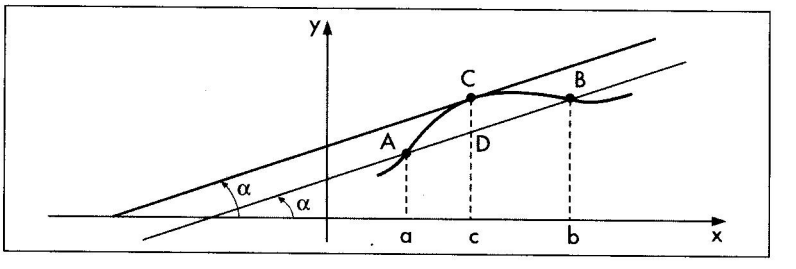
Suponhamos que f seja contínua e derivável em $]a, b[$. Então existe um ponto $c \in]a, b[$ tal que:

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Demonstração

Seja $g(x)$ a função cujo gráfico é a reta AB da Figura B.3, e $h(x) = f(x) - g(x)$.

Figura B.3: Ilustração do teorema do valor médio.



Então $h(a) = f(a) - g(a) = 0$ e $h(b) = f(b) - g(b) = 0$. Assim, pelo Teorema de Rolle existe $c \in]a, b[$ tal que $h'(c) = f'(c) - g'(c) = 0$.

Mas $g'(c)$ é igual ao coeficiente angular da reta AB, que vale $\dfrac{f(b) - f(a)}{b - a}$.

Portanto

$$f'(c) = g'(c) = \frac{f(b) - f(a)}{b - a}.$$

Teorema B.5 (Máximos e mínimos por meio da segunda derivada)

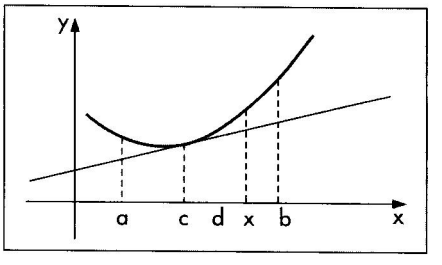
Suponhamos que f seja uma função contínua em $[a, b]$, a sua derivada f' seja contínua em $]a, b[$, e a segunda derivada f'' seja definida em $]a, b[$. Então

a) Se $f''(x) > 0$ para $x \in]a, b[$, o gráfico de f é côncavo para cima em $[a, b]$;
b) Se $f''(x) < 0$ para $x \in]a, b[$, o gráfico de f é côncavo para baixo em $[a, b]$.

Demonstração

Vejamos o caso (a). O caso (b) tem demonstração análoga.
Seja $c \in]a, b[$; a equação da reta tangente ao gráfico de f por c é dada na Figura B.4

Figura B.4: Ilustração do teorema 5.



A equação dessa reta é $y = f(c) + f'(c) \cdot (x - c)$. Precisamos provar que

$$f(x) \geqslant f(c) + f'(c) \cdot (x - c), \tag{B.3}$$

para todo $x \in [a, b]$.

Se $x = c \Rightarrow f(x) \geqslant f(c)$, a relação (B.3) é válida.

Se $x > c$, pelo teorema de valor médio, existe um ponto $d \in]c, x[$ tal que

$$\frac{f(x) - f(c)}{x - c} = f'(d),$$

ou seja,

$$f(x) - f(c) = f'(d) \cdot (x - c). \tag{B.4}$$

Como $f''(x) > 0$ para todo $x \in]a, b[$, f' é crescente em $]a, b[$. Logo, para $x > c$ vem que $f'(d) > f'(c)$ do que decorre

$$f'(d) \cdot (x - c) > f'(c) \cdot (x - c). \tag{B.5}$$

De (B.4) e (B.5) vem que

$$f(x) - f(c) > f'(c) \cdot (x - c),$$

que é a relação (B.3) que queríamos demonstrar. A mesma coisa vale para $x < c$.

Corolário B.1

Sejam f, f', f'' contínuas em $[a, b]$ e $c \in [a, b]$ com $f'(c) = 0$. Se $f''(c) > 0$, c é ponto de mínimo e, se $f''(c) < 0$, c é ponto de máximo de f.

BIBLIOGRAFIA

ALLEN, R. G. D. *Análise matemática para economistas*. Rio de Janeiro: Fundo de Cultura, 1960. v. 1 e 2.

ANTON, H. *Calculus with analytic geometry*. 5. ed. New York: John Wiley & Sons, 1995.

ANTONY, M.; BIGGS, N. *Mathematics for economics and finance*. Cambridge: University Press, 1996.

ARYA, J. C.; LARDNER, R. W. *Mathematical analysis for business, economics, and life and social sciences*. 4. ed. New Jersey: Prentice Hall, 1993.

BARNETT, R. A.; ZIEGLER, M. R.; BYLEEN, E. K. *Calculus for business, economics, life sciences, and social science*. 8. ed. New Jersey: Prentice Hall, 1999.

BOULOS, P.; ABUD, Z. I. *Cálculo diferencial e integral*. São Paulo: Makron Books, 2000. v. 1 e 2.

BRADLEY, T.; PATTON, P. *Essential mathematics for economics and business*. West Sussex: John Wiley & Sons, 1998.

CHIANG, A. *Matemática para economistas*. São Paulo: McGraw-Hill do Brasil, 1982.

GOLDSTEIN, L. J.; LAY, D. C.; SCHNEIDER, D. I. *Matemática aplicada*. 8. ed. Porto Alegre: Bookman, 2000.

GUIDORIZZI, H. *Um curso de cálculo*. 4. ed. Rio de Janeiro: LTC, 2000. v. 1 e 2.

HAEUSSLER E. F. J.; PAUL, R. S. *Introductory mathematical analysis for business, economics, and life and social sciences*. 9. ed. New Jersey: Prentice Hall, 1999.

HIRSCHEY, M.; PAPPAS, J. L. *Managerial economics*. 8. ed. Orlando: Harcourt Brace College Publishers, 1996.

HOFFMANN, L. D.; BRADLEY, G. L. *Cálculo*: um curso moderno e suas aplicações. 6. ed. Rio de Janeiro: LTC, 1999.

IEZZI, G.; HAZZAN, S. *Fundamentos de matemática elementar*. São Paulo: Atual Editora, 1978.

JACQUES, I. *Mathematics for economics and business*. 3. ed. Essex: Addison Wesley, 1999.

JAMES, D. E.; THROSBY, C. D. *Métodos quantitativos aplicados à economia*. São Paulo: Editora Atlas, 1977.

LARSON, R.; EDWARDS, B. H. *Calculus*: an applied approach. 5. ed. New York: Houghton Mifflin, 1999.

———. *Elementary linear algebra*. 4. ed. New York: Houghton Mifflin Co., 2000.

LEITHOLD, L. *O cálculo com geometria analítica*. São Paulo: Editora Harper Row do Brasil, 1977. v. 1.

MACHADO, N. J. *Cálculo*: funções de mais de uma variável. São Paulo: Atual Editora, 1979.

MANKIW, N. G. *Macroeconomia*. 3. ed. Rio de Janeiro: LTC, 1997.

PFITZNER, S. A. *Mathematical fundamentals for microeconomics*. Oxford: Blackwell Publishers, 1996.

SANDRONI, P. *Novo dicionário de economia*. 9. ed. São Paulo: Editora Best Seller, 1994.

SIMONSEN, M. H. *Teoria microeconômica*. Rio de Janeiro: Editora FGV, 1979. v. 1 e 2.

SYDSAETER, K.; HAMMOND, P. J. *Mathematics for economic analysis*. New Jersey: Prentice-Hall, 1995.

TAN, S. T. *Matemática aplicada à administração e economia*. 5. ed. São Paulo: Editora Pioneira, 2001.

VARIAN, H. R. *Microeconomia*. 5. ed. Rio de Janeiro: Editora Campus, 2000.

VICECONTI, P. E. V.; NEVES, S. *Introdução à economia*. 3. ed. São Paulo: Editora Frase, 1999.

WATSON, D. S.; HOLMAN, M. A. *Microeconomia*. São Paulo: Editora Saraiva, 1980.

WEBER, J. *Matemática para economia e administração*. 2. ed. São Paulo: Editora Harbra, 1986.

WISNIEWSKI, M. *Introductory mathematical methods in economics*. 2. ed. London: McGraw-Hill, 1996.

ZILL, D. G.; CULLEN, M. R. *Equações diferenciais*. 3. ed. São Paulo: Makron Books, 2001. v. 1.

RESPOSTAS

Capítulo 1

1. a) $a \in A$ b) $A \subset B$ c) $A \supset B$ d) $A \not\subset B$ e) $A \not\supset B$ f) $a \notin A$

2. a) $\{8, 9, 10, 11, 12\}$
   b) $\{a, e, i, o, u\}$
   c) $\{2, 4, 6, 8, 10, 12, 14, 16\}$
   d) $\{2\}$
   e) $\left\{\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}\right\}$
   f) $\{1, -1\}$
   g) $\{A, R\}$
   h) $\{-3\}$
   i) $\{1, 2, 3, 4\}$

3. a) $\{x \in N^* \mid x \leqslant 15$ e x é ímpar$\}$
   b) $\{x \in N^* \mid x = 1$ ou $x = 7\}$
   c) $\{x \mid x$ é par e $5 < x < 21\}$
   d) $\{x \in R \mid -1 \leqslant x < 10\}$

4. a) $\{5, 7, 9, 11, 12\}$
   b) $\{7\}$
   c) $\{3, 5, 7, 11\}$
   d) $\{5, 7\}$
   e) $\{3, 5, 7, 9, 12\}$

5. a e d são verdadeiras.

6. a) $\{0\}, \{1\}, \{2\}$;
   $\{0, 1\}, \{0, 2\}, \{1, 2\}$;
   $\{0, 1, 2\}$ e $\varnothing$
   b) $\{1\}, \{\{2, 3\}\}$;
   $\{1, \{2, 3\}\}$ e $\varnothing$
   c) $\{R\}, \{O\}, \{M\}, \{A\}$;
   $\{R, O\}, \{R, M\}, \{R, A\}, \{O, M\}, \{O, A\}, \{M, A\}$;
   $\{R, O, M\}, \{R, O, A\}, \{R, M, A\}, \{O, M, A\}$;
   $\{R, O, M, A\}$ e $\varnothing$

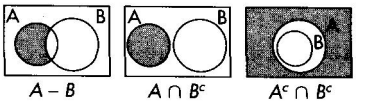
7. a) V b) V c) V

8. São todas corretas.

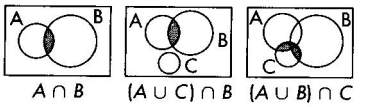
9. a) infinito
   b) infinito
   c) finito
   d) finito
   e) infinito

10. a) $\{1, 2, 3, 4, 5, 7, 9\}$
    b) $\{1, 2, 3, 4, 5, 6, 8\}$
    c) $\varnothing$
    d) $\{1, 3, 5\}$
    e) $\{7, 9\}$
    f) $\{2, 4\}$
    g) $\{1, 3, 5, 7, 9\}$
    h) $\{2, 4, 6, 8\}$
    i) $\{2, 4, 6, 8\}$
    j) $\{6, 7, 8, 9\}$
    k) $\varnothing$
    l) $\{2, 4, 6, 7, 8, 9\}$
    m) $\{2, 4, 6, 8\}$
    n) $\{1, 2, 3, 4, 5, 6, 8\}$
    o) $\{1, 3, 5\}$
    p) $\{6, 7, 8, 9\}$

11. a) $A \cap B = \varnothing$
    b) $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 9\}$
    c) $B \cap C = \varnothing$
    d) $A^c \cap B^c = \{8, 10\}$
    e) $\{1, 3, 5, 7, 9\}$
    f) $A \cup \varnothing = A$
    g) $\varnothing$
    h) $\{1, 3, 5, 7\}$

12. 

$A - B$ $A \cap B^c$ $A^c \cap B^c$

13. 

$A \cap B$ $(A \cup C) \cap B$ $(A \cup B) \cap C$

14. a) E
    b) A
    c) E
    d) A
    e) $\varnothing$
    f) E
    g) $\varnothing$
    h) A
    i) A^c
    j) A
    k) A
    l) $\varnothing$

18. a) $A \cap B$
    b) A
    c) A
    d) $\varnothing$

19. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

20. a) $\{(0, 2), (0, 3), (1, 2), (1, 3)\}$
    b) $\{(a, x), (a, y), (a, z), (b, x), (b, y), (b, z), (c, x), (c, y), (c, z)\}$
    c) $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

21. 15 maneiras diferentes.

22. a) $\{(1, 1), (1, 2), (1, 5), (1, 7), (1, 8), (2, 1), (2, 2), (2, 5), (2, 7), (2, 8), (5, 1), (5, 2), (5, 5), (5, 7), (5, 8), (7, 1), (7, 2), (7, 5), (7, 7), (7, 8), (8, 1), (8, 2), (8, 5), (8, 7), (8, 8)\}$

b) $\{(1, 2), (1, 5), (1, 7), (1, 8), (2, 5), (2, 7), (2, 8), (5, 7), (5, 8), (7, 8)\}$

c) $\{(1, 5), (2, 7)\}$

d) $\{(5, 1)\}$

23. Se $A = \{1, 2, 3, 4, 5, 6\}$, o conjunto dos resultados possíveis será:

$A \times A = A^2 = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2) (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$.

24. $\{(K, K), (K, C), (C, K), (C, C)\}$, em que K e C representam cara e coroa, respectivamente.

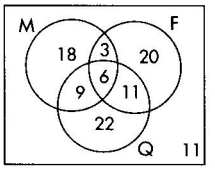
25. b) $E - A$ c) E d) A

26. $n = 10$

27. a) $A_1 = \{1\}; A_2 = \{2\}; A_3 = \{3, 4, 5, 6\}$

b) $A_1 = \{1, 2\}; A_2 = \{3, 4\}; A_3 = \{5, 6\}$

28. a)

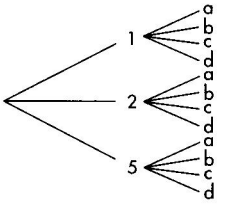


b) 29

c) 36, 40 e 48

d) 64 e 89

29.



30. 1: $A \cap B \cap C$

2: $A \cap C - A \cap B \cap C$

3: $A \cap B - A \cap B \cap C$

4: $B \cap C - A \cap B \cap C$

5: $A - (A \cap B) \cup (A \cap C)$

6: $B - (A \cap B) \cup (B \cap C)$

7: $C - (A \cap C) \cup (B \cap C)$

31. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

32. 170

33. a) 2% b) 41%

34. Não

Capítulo 2

1. a) F
b) F
c) F
d) F
e) V
f) F
g) V
h) V
i) F
j) F

2. a) 0,4
b) 1,666...
c) 1,4
d) 0,32
e) 0,2525...
f) 0,4666...

3. a) 1,28
b) 0,28
c) 0,63
d) 1,07
e) 1,52
f) 0,15

4. a) $\frac{43}{100}$
b) $\frac{7}{100}$
c) $\frac{2.454}{1.000}$
d) $\frac{1.212}{100}$
e) $-\frac{72}{100}$
f) $\frac{31.415}{10.000}$

5. a) $\frac{8}{9}$ c) $\frac{23}{9}$ e) $\frac{59}{90}$
   b) $\frac{24}{99}$ d) $\frac{65}{90}$ f) $\frac{563}{900}$

6. $x = 0$ e $y = 0$

7. a) 3,4641 b) 5,4772 c) 8,8318 d) 22,3607

8. a) $S = \{16\}$ e) $S = \left\{\frac{17}{3}\right\}$ h) $S = \{1, 5\}$
   b) $S = \{7\}$ f) $S = \{0\}$ i) $S = \{14\}$
   c) $S = \{3\}$ g) $S = \{0\}$ j) $S = \left\{\frac{81}{7}\right\}$
   d) $S = \{3\}$

9. a) $S = \left\{\frac{5}{7}\right\}$ h) $S = \left\{\frac{-6}{5}\right\}$
   b) $S = \left\{\frac{67}{8}\right\}$ i) $S = \{-2\}$
   c) $S = \left\{\frac{14}{5}\right\}$ j) $S = \{-10\}$
   d) $S = \{-1\}$ k) $S = \left\{\frac{33}{14}\right\}$
   e) $S = \left\{\frac{36}{53}\right\}$ l) $S = \left\{\frac{M - 100}{100}\right\}$
   f) $S = \left\{\frac{28}{15}\right\}$ m) $S = \left\{\frac{18K - 29}{2}\right\}$
   g) $S = \left\{\frac{31}{44}\right\}$ n) $S = \left\{\frac{1 + 3y}{y - 2}\right\}, y \neq 2$

10. 140 unidades

11. 200 camisas

12. 50 meses

13. a) $S = \{x \in R \mid x > 5\}$
    b) $S = \{x \in R \mid x > -4\}$
    c) $S = \{x \in R \mid x \geqslant -6\}$
    d) $S = \{x \in R \mid x \leqslant 0\}$
    e) $S = \left\{x \in R \mid x > \frac{5}{3}\right\}$
    f) $S = \left\{x \in R \mid x \geqslant \frac{27}{5}\right\}$
    g) $S = \{x \in R \mid x \leqslant -7\}$
    h) $S = \left\{y \in R \mid y \geqslant \frac{43}{11}\right\}$
    i) $S = \left\{m \in R \mid m \leqslant \frac{5}{2}\right\}$

14. 500 unidades

15. entre 180 e 380 unidades

16. a) $S = \{1, 4\}$ g) $S = \{0, 5\}$
    b) $S = \{3, 4\}$ h) $S = \{3 + 2\sqrt{3}, 3 - 2\sqrt{3}\}$
    c) $S = \{2, 4\}$ i) $S = \{1 + \sqrt{6}, 1 - \sqrt{6}\}$
    d) $S = \{2\}$ j) $S = \varnothing$
    e) $S = \varnothing$ k) $S = \{2, -6\}$
    f) $S = \{1, 2\}$ l) $S = \varnothing$

17. a) $S = \{0, 5\}$ d) $S = \{4, -4\}$
    b) $S = \{0, 3\}$ e) $S = \{2, -2\}$
    c) $S = \{5, -5\}$ f) $S = \{0\}$

18. –9/2

19. $\frac{3 + \sqrt{5}}{2}$ ou $\frac{3 - \sqrt{5}}{2}$

20. $x = 2$ ou $x = 8$

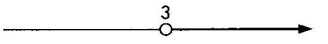
21. $x = 5$

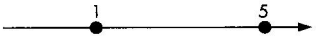
22. \$ 5,00 ou \$ 15,00

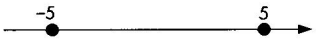
23. a) [2, 20] c) [2, 7[
    b) [7, 8] d)]–∞, 2[∪]8, ∞[

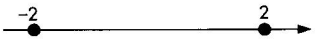
24. a) [1, 5[b) [0, ∞[c) [5, ∞[

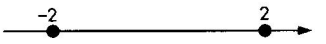
25. a)

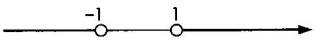

    b)


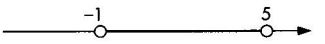
    c)


    d)


    e)


    f)


    g)


    h)


26. a) $-12 < x < 12$ d) $x > 8$ ou $x < -8$
    b) $3 < x < 9$ e) $x > 9$ ou $x < 5$
    c) $4 > x > -3$ f) $x < -1$ ou $x > 7/3$

27. $16{,}16 < x < 31{,}84$

28. $10{,}05 < x < 19{,}95$

Capítulo 3

1. a) $\{(1,3), (1, 5), (1, 8), (1, 9), (3, 5), (3, 8), (3, 9), (5, 8), (5, 9), (7, 8), (7, 9)\}$
   b) $\{(3, 3), (5, 3), (5, 5), (7, 3), (7, 5)\}$
   c) $\{(1, 3), (1, 5), (1, 8), (1, 9), (3, 3), (3, 9), (5, 5)\}$
   d) $\{(3, 3), (5, 5)\}$
   e) $\{(1, 3), (3, 5), (7, 9)\}$

2. diagrama de flechas da resposta anterior.

3. a) $D = \{1, 3, 5, 7\}$, $Im = \{3, 5, 8, 9\}$
   b) $D = \{3, 5, 7\}$, $Im = \{3, 5\}$
   c) $D = \{1, 3, 5\}$, $Im = \{3, 5, 8, 9\}$
   d) $D = \{3, 5\}$, $Im = \{3, 5\}$
   e) $D = \{1, 3, 7\}$, $Im = \{3, 5, 9\}$

4. a) 11
   b) 39
   c) -3
   d) -10
   e) $7\sqrt{2} - 3$
   f) $1/2$
   g) $-16/3$
   h) $7(a + b) - 3$

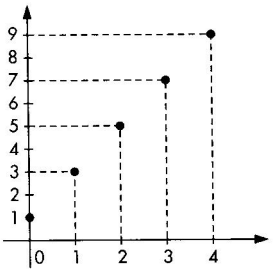
5. a) 3
   b) -11
   c) 26
   d) $-7/2$

6. a) x_0^2 b) $(x_0 + h)^2$ c) $2x_0h + h^2$

7. 1 e 3

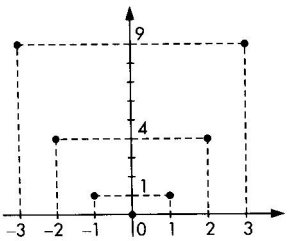
8. $m = 3$

9.



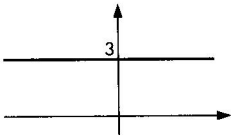
$Im(f) = \{1, 3, 5, 7, 9\}$

10.

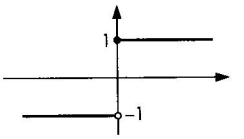


$Im(f) = \{0, 1, 4, 9\}$

11.



12.



13. a) $R(x) = 5x$
    b) \$ 200,00
    c) 140 unidades

14. a) \$ 120,00
    b) \$ 2,00

15. a) 4.333,33
    b) 234,33

16. a) \$ 29,00
    b) 16,50
    c) \$ 4,00

17. a) \$ 60,00
    b) \$ 150,00
    c) $\begin{cases} y = 0{,}1x \text{ para } x \leqslant 900 \\ y = 90 + 0{,}2(x - 900) \\ \text{para } x \geqslant 900 \end{cases}$

18. a) \$ 8,00
    b) \$ 13,00
    c) \$ 41,80
    d) $\begin{cases} y = 8 \text{ para } x \leqslant 10 \\ y = 8 + 1(x - 10) \text{ para } 10 \leqslant x \leqslant 20 \\ y = 18 + 1{,}4(x - 20) \text{ para } x \geqslant 20 \end{cases}$

19. $S(x) = 2\,000 + 50x$

20. $A = x \cdot (20 - x)$

21. $A = \dfrac{\sqrt{3}}{4} \cdot x^2$

22. São funções: a, c, d, f, i, j, l.

23. a) R
    b) $R - \{2\}$
    c) $R - \{0, 3\}$
    d) $[0, \infty[$
    e) $[2, \infty[$
    f) $]-\infty, 2]$
    g) $]1, \infty[$
    h) $[3, \infty[$
    i) $[3, \infty[$
    j) $[2, \infty[$

24. Crescente: [–7, –4], [–1, 6]
Decrescente: [–4, –1], [6, 7]
Pontos de máximo: –4 e 6
Pontos de mínimo: –7, –1 e 7

25. a) ⊖ 3 ⊕

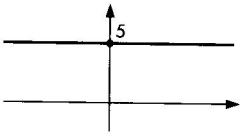
b) ⊕ 4 ⊖

c) ⊕ 2 ⊖ 5 ⊕

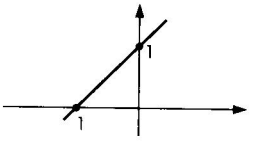
d) ⊖ 2 ⊕ 5 ⊖

e) ⊕ –1 ⊖ 3 ⊕ 4 ⊖ 5 ⊕

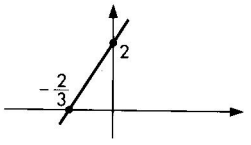
26. a)



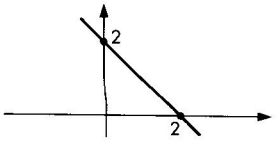
b)



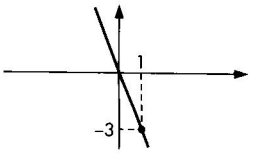
c)



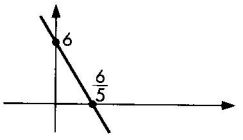
d)



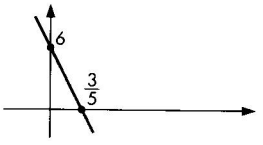
e)



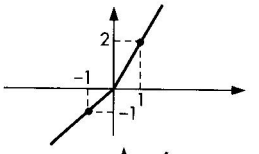
f)



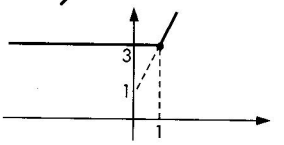
g)



h)



i)



27. a) ⊖ 3 ⊕

b) ⊖ –4 ⊕

c) ⊕ 4 ⊖

d) ⊕ 0 ⊖

e) ⊖ $-\frac{2}{5}$ ⊕

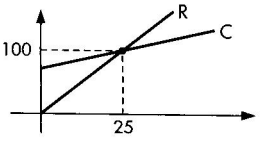
28. a) 5
b) 1
c) 1/4
d) –3/7

29. a) $y = 2x + 1$
b) $y = 3x$
c) $y = -x + 3$
d) $y = 2x$
e) $y = -3x - 4$
f) $y = -x - 2$

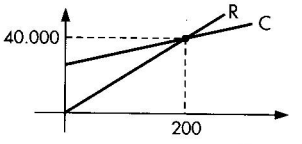
30. a) $y = x + 1$
b) $y = \frac{2}{5}x + \frac{2}{5}$
c) $y = -\frac{3}{2}x + 4$

31. a) $y = -\frac{3}{4}x + 3$
b) $y = \frac{1}{2}x + 2$
c) $y = -x + 5$

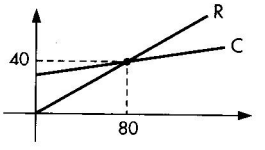
32. a)



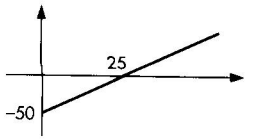
b)



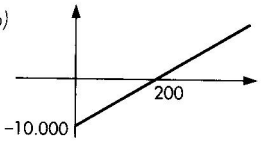
c)



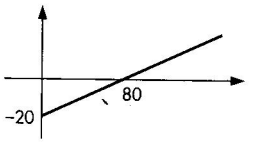
33. a)



b)



c)



34. 500 unidades

35. 900 unidades

36. a) 500 unidades
b) 400 unidades
c) 75%

37. a) $L = 2x - 30\,000$ b) $LL = 1{,}4x - 21\,000$

38. 807,7 unidades

39. a) $R = 10x$
b) $C = 150 + 7x$
c) 50
d) $L = 3x - 150$
e) 110

40. a) 227,3 unidades
b) \$ 11,00
c) \$ 8.500,00
d) 64,7%

41. a) \$ 200,00 b) 300/7

42. \$ 27,50

43. 500 unidades

44. Não é vantajosa.

45. 2 horas

46. 100 km

47. a) $C = 2.000 + 24x$ b) \$ 9.200,00

48. $C = 6.000 + 60x$

49. \$ 25,00

50. a) $p = 1{,}25\,c$ b) 25%

51. 42,86%

52. 20%

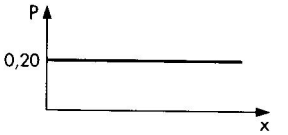
53. Demonstração.

54. $m_p = \dfrac{m_c}{1 + m_c}$

55. $p = -0{,}2x + 30$

56. $p = -0{,}01x + 7$

57.



58. $p = 0{,}004x - 10$

59. $p = 300 + 0{,}5 \cdot x$

60. a) demanda
b) oferta
c) oferta
d) demanda
e) oferta

61. a) \$ 15,00 b) \$ 42,50

62. a) \$ 2,00
b) 600 ton
c) $x = 650$ ton e $p = 3{,}5$

63. a) \$ 14,00
b) 25 unidades
c) \$ 12,00

64. 182,2

65. 20 unidades

66. a) $p = 60 - 2x$; $p = 70 - 2x$; $p = 80 - 2x$.
b) Desloca-se paralelamente para cima.

67. a) $p = 41 + 0{,}5x$ b) $p = 39 + 0{,}5x$

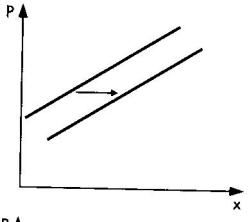
68. a) \$ 70,00
b) \$ 73,00
c) \$ 162,00

69. \$ 58,00

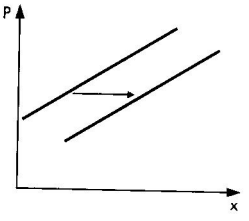
70. a) \$ 40,00 b) \$ 43,38
Sugestão: faça, por exemplo, $p = 100$ na função oferta; o valor de x será 160. Portanto, na nova curva de oferta, teremos $p = 110$ (10% a mais) e $x = 160$.
Obtenha, de modo análogo, outro ponto da curva de oferta e ache a equação da reta de oferta.

71. a) \$ 40,00 b) \$ 46,67

72. a)



b)



73. a) \$ 1.400,00
b) \$ 600,00
c) 10 anos

74. a) \$ 6.000,00
b) \$ 3.500,00

75. \$ 18.333,33

76. a) $V = 10.000 - \frac{4.000}{3} \cdot x$
b) $\frac{16.000}{3}$

77. 7,5 anos

78. a) $C = 800 + 0,8y$
b) $S = -800 + 0,2y$

79. a) $S = 0,4y - 500$
b) \$ 1.250,00

80. a) $C = 800 + 0,65y$
b) \$ 1.000,00

81. \$ 500,00

82. \$ 750,00

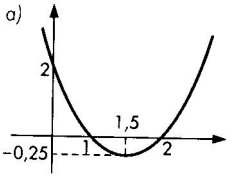
83. \$ 60,00

84. \$ 85,00

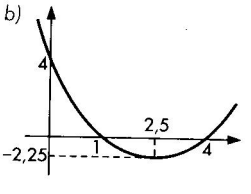
85. $C = 0,6y + 2.000$

86. $S = 0,4y - 2.000$

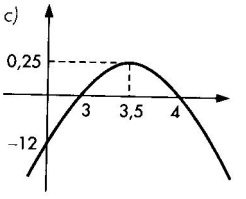
87. a)



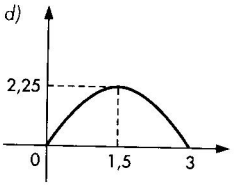
b)



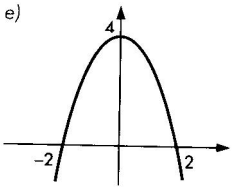
c)



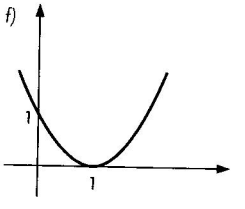
d)



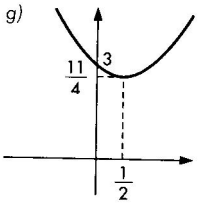
e)



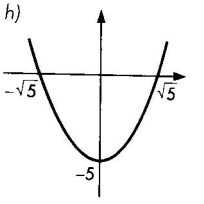
f)



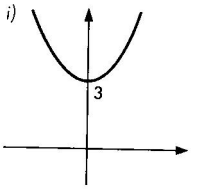
g)



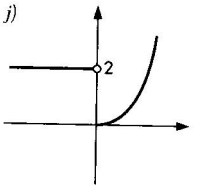
h)



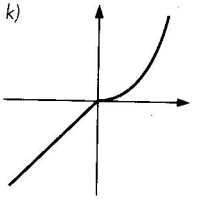
i)



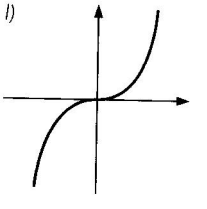
j)



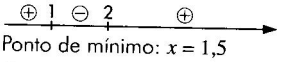
k)



l)

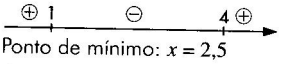


88. a)



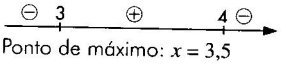
Ponto de mínimo: $x = 1,5$
Conjunto-imagem: $[-0,25, \infty[$

b)



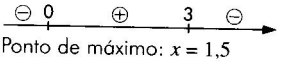
Ponto de mínimo: $x = 2,5$
Conjunto-imagem: $[-2,25, \infty[$

c)



Ponto de máximo: $x = 3,5$
Conjunto-imagem: $]-\infty, 0,25]$

d)



Ponto de máximo: $x = 1,5$
Conjunto-imagem: $]-\infty, 2,25]$

e) $\ominus$ -2 $\oplus$ 2 $\ominus$

Ponto de máximo: $x = 0$

Conjunto-imagem: $]-\infty, 4]$

f) $\oplus$ 1 $\oplus$

Ponto de mínimo: $x = 1$

Conjunto-imagem: $[0, \infty[$

g) $\oplus$

Ponto de mínimo: $x = 1/2$

Conjunto-imagem: $\left[\frac{11}{4}, \infty\right[$

h) $\oplus$ $-\sqrt{5}$ $\ominus$ $\sqrt{5}$ $\oplus$

Ponto de mínimo: $x = 0$

Conjunto-imagem: $[-5, \infty[$

i) $\oplus$

Ponto de mínimo: $x = 0$

Conjunto-imagem: $[3, \infty[$

j) $\oplus$ 0 $\oplus$

Ponto de mínimo: $x = 0$

Conjunto-imagem: $[0, \infty[$

k) $\ominus$ 0 $\oplus$

Conjunto-imagem: R

l) $\ominus$ 0 $\oplus$

Conjunto-imagem: R

89. a) $y > 0$ para $1 < x < 3$ ou $x > 5$
$y < 0$ para $x < 1$ ou $3 < x < 5$
$y = 0$ para $x = 1$ ou $x = 5$

b) $y > 0$ para $x < -2$ ou $2 < x < 3$
$y < 0$ para $-2 < x < 2$ ou $y > 3$
$y = 0$ para $x = 3$

c) $y > 0$ para $x < -1$
ou $0 < x < 1$ ou $x > 3$
$y < 0$ para $1 < x < 3$ ou $-1 < x < 0$
$y = 0$ para $x = 1$ ou $x = -1$

d) $y > 0$ para $-2 < x < 2$ ou $x > 4$
$y < 0$ para $x < -2$ ou $2 < x < 4$
$y = 0$ para $x = 2$ ou $x = 4$

90. a) $\{x \in R \mid x \leqslant 0 \text{ ou } x \geqslant 6\}$
b) $\{x \in R \mid 0 \leqslant x \leqslant 3\}$
c) $\{x \in R \mid x < -2 \text{ ou } x > 2\}$
d) $\{x \in R \mid 0 < x \leqslant 3 \text{ ou } x > 6\}$
e) $\{x \in R \mid x < -4 \text{ ou } -1 \leqslant x \leqslant 1\}$

91. a) Ponto de máximo: $x = 2$
Ponto de mínimo: $x = 4$

b) Ponto de máximo: $x = 2$
Ponto de mínimo: $x = 0$

c) Pontos de máximo: $x = -1$ ou $x = 1$
Ponto de mínimo: $x = 0$

d) Ponto de máximo: $x = 5$
Ponto de mínimo: $x = 8$

92. a) $x = 5$ b) $x = 19/4$

93. a) $x = 20$ b) $x = 4{,}5$

94. a) $p = -0{,}02x + 40$ b) \$ 30,00

95. a) $p = -0{,}05x + 25$ b) \$ 12,50

96. a) $p = -0{,}015x + 13$ b) \$ 8,50

97. a) \$ 7,50 b) \$ 9,00

98. a) $p = -5x + 350$ b) \$ 175,00

99. a) \$ 60,00 b) \$ 50,00

100. a) \$ 51,00 c) \$ 40,00
b) \$ 51,00

101. a) $R = -0{,}01x^2 + 10x, \quad 0 \leqslant x \leqslant 600$
b) \$ 5,00

102. a) $p = -0{,}02x + 8$ b) \$ 4,00

103. a) \$ 51,50 b) \$ 52,50

104. a) \$ 19,00 b) \$ 12,50

105. a) \$ 21,50 b) \$ 10,00

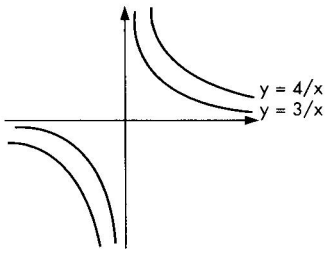
106. a) $L = -3x^2 + 180x - 200$
b) $x = 30$

107. 80/3

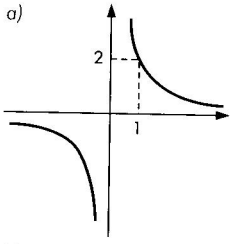
108. a) \$ 6,50 b) $6 \leqslant p \leqslant 7$

109. 3,5 unidades

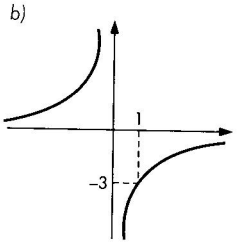
110.



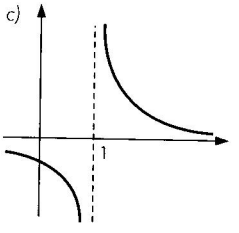
111. a)



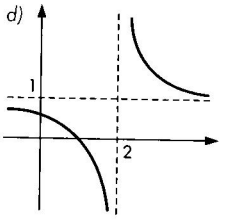
b)



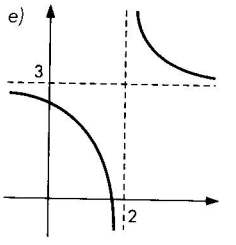
c)



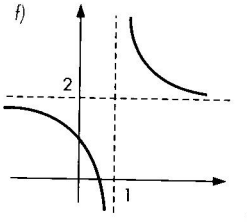
d)



e)



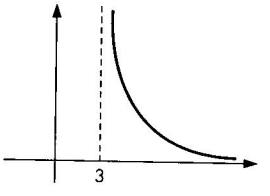
f)



112. a) $x = 1$ e $p = 10$
b) $x = 1$ e $p = 8$
c) $x = 1$ e $p = 55/7$

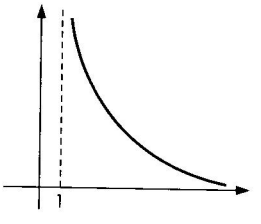
113. a) $C = \dfrac{12 \cdot 10^5}{x}$ b) \$ 3.000,00

114. a)



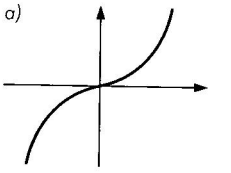
b) 2,5 bilhões c) tende a infinito

115. a)

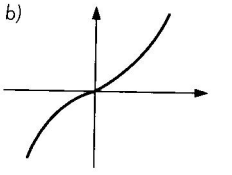


b) 2,5 bilhões c) tende a infinito

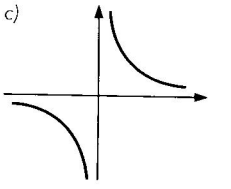
116. a)



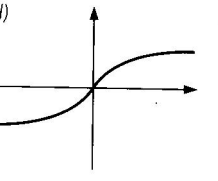
b)



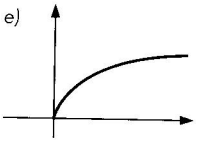
c)



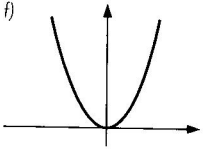
d)



e)

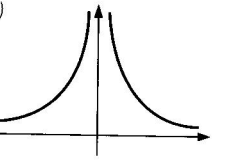


f)

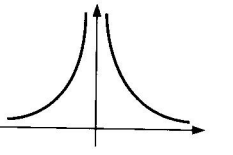


g)

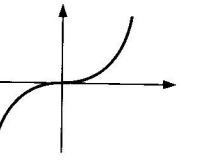
h)



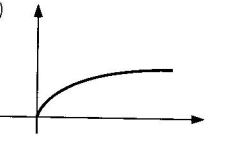
i)



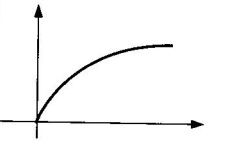
j)



k)

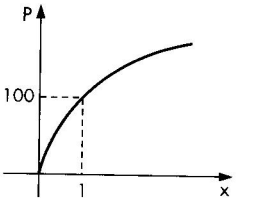


l)

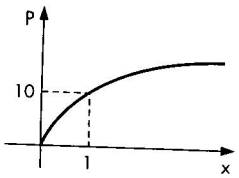


117. a) $x = 1$ e $p = 1$ b) $x = 1$ e $p = 1$

118. a) 400; 25
b) 800; 12,5
c) Dobrará.
d) 0
e)



119. a) 40; 5
b) 160; 2,5
c) Quadruplicará.
d) 0
e)



120. a) 19 b) 48 c) $ 1.698,50

121. a) 63 b) $ 1.294,96 c) $ 599,48

122. a) 16
b) 81
c) 1
d) 1/9
e) 1/8
f) 1/16
g) 1/25
h) 1/8
i) 1/81
j) 4/9
k) −8/27
l) 3/2
m) 9/4

123. a) x^5
b) x^9
c) x^4
d) x^7y^7
e) $144x^6$
f) 2
g) 4
h) 1/2
i) $1 + i$

124. a) 2
b) 5
c) 1,7321
d) 2,8284
e) 14,6969
f) 2,5119
g) 1,0188
h) 1,8286

125. a) 16
b) 6
c) 3
d) 128
e) 0,25

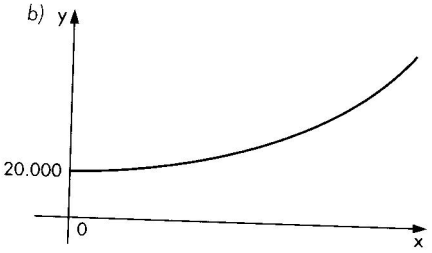
126. a) 8.867 b) 16.990

127. 3,53%

128. 2,81%

129. 765,77 bilhões de dólares

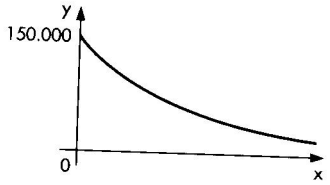
130. a) $y = 20.000(1,02)^x$
b)



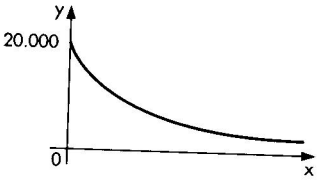
131. 25.598 habitantes

132. 2.488,32 unidades

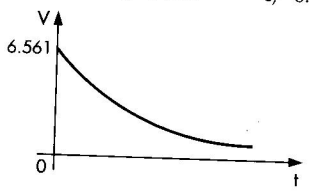
133. a) $ 110.613,62
b)



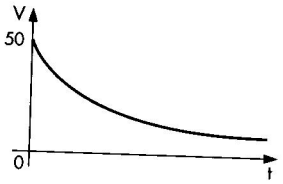
134. a) $ 8.874,11
b)



135. a) 6.561 b) $ 243 c) 6.318
d)



136. a) 50 mil dólares
b)

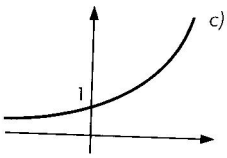


137. $K = -2,60\%$

138. $V = 4.000\,(0,866)^t$

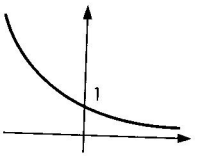
139. a) $ 25.000 b) $17.000 \cdot (0,9)^{x-2}$ para $x \geqslant 2$

140. a)



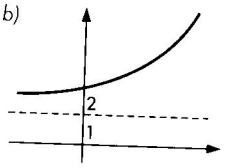
$D = R$
$Im(f) =]0, \infty[$

c)



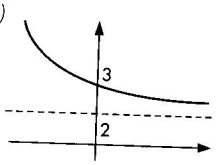
$D = R$
$Im(f) =]0, \infty[$

b)



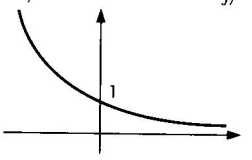
$D = R$
$Im(f) =]1, \infty[$

d)



$D = R$
$Im(f) =]2, \infty[$

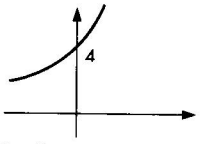
e)



$D = R$

$Im(f) =]0, \infty[$

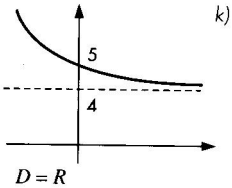
j)



$D = R$

$Im(f) =]0, \infty[$

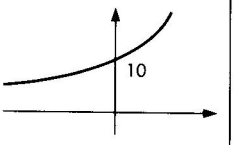
f)



$D = R$

$Im(f) =]4, \infty[$

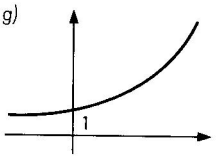
k)



$D = R$

$Im(f) =]0, \infty[$

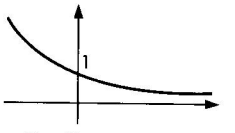
g)



$D = R$

$Im(f) =]0, \infty[$

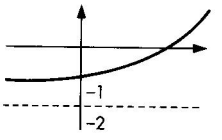
l)



$D = R$

$Im(f) =]0, \infty[$

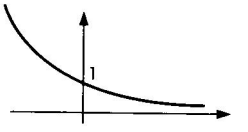
h)



$D = R$

$Im(f) =]-2, \infty[$

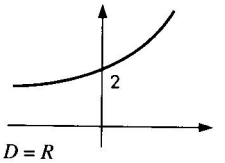
m)



$D = R$

$Im(f) =]0, \infty[$

i)



$D = R$

$Im(f) =]0, \infty[$

141. a) 3 b) 2 c) 4 d) 0 e) 1 f) 4 g) –3 h) –2 i) –2 j) –1/2 k) 6 l) 6

142. a) 1,7324 b) 0,8451 c) 2,0864 d) 1,5391 e) 3,4340 f) 1,9459 g) 0,4055 h) 5,4931 i) 0,5306 j) –0,2231 k) –0,0834 l) –0,6162 m) 2,8074 n) 1,2789 o) 0,5681

143. a) 0,78 b) 0,90 c) 1,08 d) 1,38 e) 1,30 f) 2,48 g) 0,70 h) 1,70 i) –0,70 j) –1,52

144. a) $x = 0{,}625$ b) $x = 0{,}80$ c) $x = 3{,}20$ d) $x = 1{,}15$ e) $x = 1{,}67$ f) $x = -0{,}87$

145. a) –3,9694 b) 2,5779 c) 1,4882 d) 3,4650 e) 7,8074

146. a) 1,3863 b) 0,8214 c) 0,4241 d) 0,1329

147. 23,5 anos aproximadamente

148. 22,5 anos

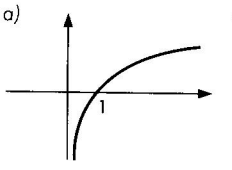
149. 22,7 anos aproximadamente

150. 4,3 anos aproximadamente

151. 3,1 anos aproximadamente

152. 15 anos

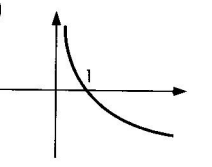
153. a)



$D =]0, \infty[$

$Im(f) = R$

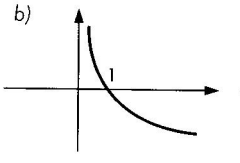
c)



$D =]0, \infty[$

$Im(f) = R$

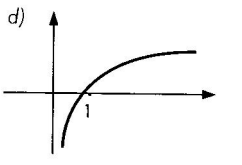
b)



$D =]0, \infty[$

$Im(f) = R$

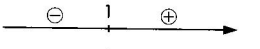
d)



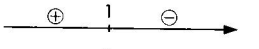
$D =]0, \infty[$

$Im(f) = R$

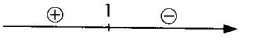
154. a)



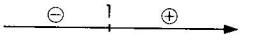
b)



c)

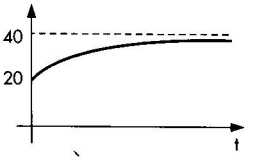


d)

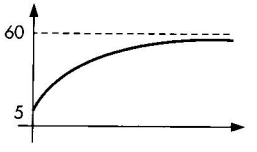


155. a) $\{x \in R \mid x > 3\}$
b) $\{x \in R \mid x < 2\}$
c) $\{x \in R \mid x < 1 \text{ ou } x > 3\}$
d) $\{x \in R \mid x < -2 \text{ ou } x > 2\}$
e) $\{x \in R \mid 0 < x < 4\}$

156. a) 20
b) 32,6
c) 40
d)



157. a) 5
b) 52,3
c) 60
d)



158. $K = 0{,}22$ e $B = 6{,}25$

159. \$ 2.147,93

160. \$ 14.282,46

161. \$ 3.313,47

162. \$ 3.814,48

163. 3,71% ao mês

164. 36,8 meses aproximadamente

165. 1,6 anos

166. \$ 5.327,83

167. \$ 8.556,29

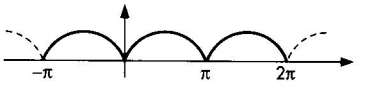
168. 10% ao mês

169. 8% ao mês

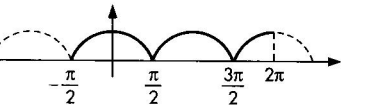
170. 5% ao mês

171. a) R\$ 279.161,54 b) R\$ 277,91

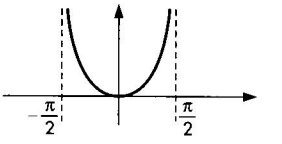
172. a)



b)



c)



173. a) 0 ⊕ π ⊖ 2π

b) 0 ⊕ π/2 ⊖ 3π/2 ⊕ 2π

c) −π/2 ⊖ 0 ⊕ π/2

d) 0 ⊖ 1 ⊕ π ⊖ 2π

e) 0 ⊖ π/2 ⊕ 3 ⊖ 3π/2 ⊕ 2π

f) −π/2 ⊖ −1 ⊕ 0 ⊖ 1 ⊕ π/2

174. a) Ponto de máximo: $\pi/2$
Pontos de mínimo: 0 e π
b) Ponto de máximo: 2π
Ponto mínimo: π
c) Ponto de mínimo: 0
d) Pontos de mínimo: 0, π e 2π
Pontos de máximo: $\pi/2$ e $3\pi/2$
e) Pontos de mínimo: $\pi/2$ e $3\pi/2$
Pontos de máximo: 0, π e 2π

Capítulo 4

1. a) $f(n) = n^2$ (divergente)
b) $f(n) = (-1)^n \cdot n$ (divergente)
c) $f(n) = 2^{n-1}$ (divergente)
d) $f(n) = 5(n-1)$ (divergente)
e) $f(n) = \left(\frac{1}{3}\right)^{n-1}$ (converge para 0)
f) $f(n) = (0{,}1)^n$ (converge para 0)

2. a) Converge para 0.
   b) Divergente.
   c) Converge para 0.
   d) Converge para 2.
   e) Converge para 0.
   f) Divergente.
   g) Converge para 1.
   h) Converge para 0.
   i) Converge para 0.
   j) Divergente.
   k) Divergente.
   l) Converge para 5.
   m) Converge para $\frac{4}{3}$.

3. $f(n)$ converge para 0 e $g(n)$ para 1/2.

4. $h(n) = \frac{n+3}{2n}$ converge para $\frac{1}{2}$.

5. $h_1(n) = \frac{n+1}{2n^2}$ converge para 0.

6. $h_2(n) = \frac{1-n}{2n}$ converge para $-\frac{1}{2}$.

7. $h_3(n) = \frac{2}{n+1}$ converge para 0.

8. a) 8; 8; 8
   b) 7; 7; 7
   c) $-\frac{5}{3}; -\frac{5}{3}; -\frac{5}{3}$
   d) –7; –7; –7
   e) 7; 7; 7
   f) 0; 0; 0
   g) 7; 4; não existe
   h) 5; 5; 5
   i) 0; 0; 0
   j) $\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2}$
   k) 0; 0; 0

9. a) 6
   b) 14
   c) 1/10
   d) –1/3
   e) 0
   f) –2
   g) 1
   h) –1
   i) 0
   j) 1/4
   k) 12
   l) 27
   m) –2/3
   n) 1

10. a) ∞ e $-\infty$
    b) $-\infty$ e ∞
    c) ∞ e ∞
    d) ∞ e $-\infty$
    e) $-\infty$ e ∞
    f) ∞ e $-\infty$
    g) ∞ e $-\infty$
    h) ∞ e ∞
    i) $-\infty$ e $-\infty$
    j) ∞ e $-\infty$
    k) ∞ e ∞
    l) ∞ e $-\infty$
    m) ∞ e ∞
    n) ∞ e ∞
    o) ∞ e ∞
    p) ∞ e ∞

11. a) 0
    b) 0
    c) ∞
    d) ∞
    e) ∞
    f) $-\infty$
    g) ∞
    h) 0
    i) ∞
    j) ∞
    k) ∞
    l) $-\infty$
    m) ∞
    n) ∞
    o) $-\infty$
    p) 2
    q) 2
    r) 25/16
    s) 1/2
    t) 0
    u) 0
    v) 0
    w) ∞
    x) 1/2

12. Sim

13. Não

14. Não

15. $k = 7$

16. a) $x = -1$
    b) $y = 1$

17. a) $x = 1$
    b) não existem

18. $x = 0$

19. $y = 0$

20. a) e^2
    b) $e^{1/3}$
    c) e^2
    d) $e^{3/2}$
    e) 1

21. \$ 3.644,24

22. \$ 5.525,85

23. \$ 7.899,18

24. 6,42 anos

25. a) –1
    b) k
    c) 0
    d) k
    e) a/b
    f) a/b
    g) 0

Capítulo 5

1. a) 8
   b) 2
   c) −3
   d) 1
   e) 0
   f) −1/4
   g) −1/25
   h) 9

2. a) $f'(x) = 2x$
   b) $f'(x) = 2$
   c) $f'(x) = -3$
   d) $f'(x) = 2x - 3$
   e) $f'(x) = 2x$
   f) $f'(x) = -1/x^2$
   g) $f'(x) = -1/x^2$
   h) $f'(x) = 2x - 3$

3. Demonstração.

4. Demonstração.

5. a) $f'(x) = 0$
   b) $f'(x) = 5x^4$
   c) $f'(x) = 50x^4$
   d) $f'(x) = x$
   e) $f'(x) = 2x + 3x^2$
   f) $f'(x) = 30x^2 + 10x$
   g) $f'(x) = 2$
   h) $f'(t) = 6t - 6$
   i) $f'(u) = 15u^2 - 4u + 6$
   j) $f'(x) = \dfrac{3}{x}$
   k) $f'(x) = \dfrac{10}{x} - 3$
   l) $f'(x) = 5\cos x - 2\,\text{sen}\, x$
   m) $f'(x) = \text{sen}\, x + x \cdot \cos x$
   n) $f'(x) = 2x \ln x + x$
   o) $f'(x) = (4x - 3)(2x - 1) + (2x^2 - 3x + 5) \cdot 2$
   p) $f'(x) = \dfrac{x^2 \cdot \cos x - 2x \cdot \text{sen}\, x}{x^4}$
   q) $f'(x) = 1/\cos^2 x$
   r) $f'(x) = \dfrac{-1}{(x-2)^2}$
   s) $f'(x) = -6x^{-4} - 10x^{-3}$
   t) $f'(x) = \dfrac{2}{3}\, x^{-1/3}$
   u) $f'(x) = \dfrac{1}{3}\, x^{-2/3} + \dfrac{1}{4}\, x^{-3/4}$
   v) $f'(x) = \dfrac{3}{2}\, x^{-1/2} + \dfrac{5}{3}\, x^{-2/3}$
   w) $f'(x) = \dfrac{1}{2}\, x^{-1/2}\, \text{sen}\, x + x^{1/2} \cdot \cos x$
   x) $f'(x) = x^{-3/2} - \dfrac{1}{2}\, x^{-3/2} \cdot \ln x$

6. a) $f'(x) = 6(2x - 1)^2$
   b) $f'(x) = 8(2x - 1)^3$
   c) $f'(x) = 6(5x^2 - 3x + 5)^5 \cdot (10x - 3)$
   d) $f'(x) = 3\left(\dfrac{1}{x^2} + \dfrac{1}{x} + 1\right)^2 \cdot \left(-\dfrac{2}{x^3} - \dfrac{1}{x^2}\right)$
   e) $f'(x) = -5(x^2 - 3x - 2)^{-6} \cdot (2x - 3)$
   f) $f'(x) = \dfrac{6x - 2}{3x^2 - 2x}$
   g) $f'(x) = \dfrac{2x - 3}{x^2 - 3x + 6}$
   h) $f'(x) = (2x - 3) \cdot \cos(x^2 - 3x)$
   i) $f'(x) = 2^x \cdot \ln 2$
   j) $f'(x) = 5^x \cdot \ln 5$
   k) $f'(x) = e^x + 3^x \cdot \ln 3$
   l) $f'(x) = (2x - 2)\, e^{x^2 - 2x + 1}$
   m) $f'(x) = 2x \cdot 3^{x^2 - 4} \cdot \ln 3$
   n) $f'(x) = \dfrac{2}{(x+1)^2}\, e^{x - 1/x + 1}$
   o) $f'(x) = e^x - e^{-x}$
   p) $f'(x) = \dfrac{-4}{(e^x - e^{-x})^2}$
   q) $f'(x) = (2x + 1)^{-1/2}$
   r) $f'(x) = \dfrac{2}{3}\,(2x + 1)^{-2/3}$
   s) $f'(x) = \dfrac{3}{2}\,(6x^2 + 2x + 1)^{1/2} \cdot (12x + 2)$
   t) $f'(x) = \dfrac{1}{2}(x + 1)^{-1/2} + (x^2 - 3x + 1)^{-2/3} \cdot (2x - 3) \cdot \dfrac{1}{3}$
   u) $f'(x) = \dfrac{1}{2} \cdot x^{-1/2} + \dfrac{1}{2}\,(x + 1)^{-1/2}$
   v) $f'(x) = \dfrac{1}{2}\left(\dfrac{\ln x}{e^x}\right)^{-1/2} \cdot \dfrac{\dfrac{1}{x} - \ln x}{e^x}$
   w) $f'(x) = -\dfrac{5}{2}\left(\dfrac{x + 1}{3x - 2}\right)^{-1/2} \cdot \dfrac{1}{(3x - 2)^2}$
   x) $f'(x) = 3x \cdot (3x^2 + 1)^{-1}$

7. a) $f'(x) = x^{x^2}\,(2x \ln x + x)$
   b) $f'(x) = (x^2 + 1)^x \left[\ln(x^2 + 1) + \dfrac{2x^2}{x^2 + 1}\right]$
   c) $f'(x) = \left(\dfrac{2}{x} \ln x\right) x^{\ln x}$

8. $Df^{-1}(y) = -\dfrac{1}{\sqrt{1 - y^2}}$

9. $Df^{-1}(y) = \dfrac{1}{1 + y^2}$

10. a) $f'(x) = \dfrac{3}{\sqrt{1-(3x-5)^2}}$

b) $f'(x) = \dfrac{-1}{4 \cdot \sqrt{1-\dfrac{x^2}{16}}}$

c) $f'(x) = \dfrac{2x}{1+(x^2-5)^2}$

11. Demonstração.

12. a) $y - 25 = 10(x-5)$
b) $y + 4 = -3(x-1)$
c) $y = 2x + 3$
d) $y = -(x-2)$
e) $y - 1 = \dfrac{1}{e}(x-e)$
f) $y - \dfrac{1}{3} = \dfrac{1}{9}(x-3)$
g) $y - \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}}{2}\left(x - \dfrac{\pi}{4}\right)$
h) $y - \dfrac{1}{e} = \dfrac{-2}{e}(x-1)$

13. a) 0,4
b) 0,01
c) 0,1
d) $(\ln a) \cdot d$
e) 0
f) $-\dfrac{\sqrt{3}}{4}$

14. Demonstração.

15. a) 1, 1e
b) 0,06

16. \$ 52,5

17. \$ 64

18. \$ –24

19. 1.666,67

20. \$ 400

21. \$ 50,00

22. a) $0{,}9x^2 - 5x + 20$
b) \$ 17,50
c) \$ 60,00

23. a) $0{,}2x + 5$
b) \$ 6,00
c) \$ 7,00

24. \$ 100,00

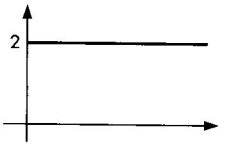
25. a) $-8x + 500$
b) \$ 420,00
c) \$ 340,00

26. $20 - 4x$

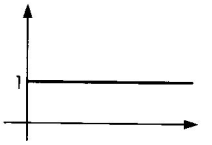
27. $\dfrac{15.000}{(x+30)^2} - 10$

28. $a - 2bx$

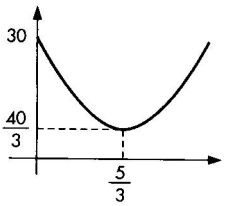
29. a) $C_{mg}(x) = 2$



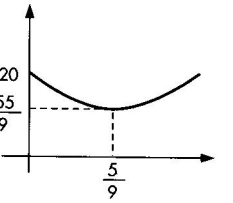
b) $C_{mg}(x) = 1$



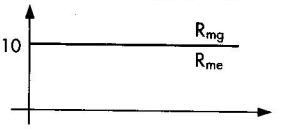
c) $C_{mg}(x) = 6x^2 - 20x + 30$



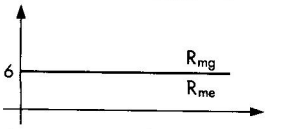
d) $C_{mg}(x) = 9x^2 - 10x + 20$



30. a) Receita marginal: $R_{mg}(x) = 10$
Receita média: $R_{me}(x) = 10$



b) Receita marginal: $R_{mg}(x) = 6$
Receita média: $R_{me}(x) = 6$

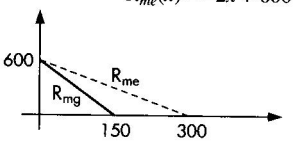


c) Receita marginal:

$$R_{mg}(x) = -4x + 600$$

Receita média:

$$R_{me}(x) = -2x + 600$$

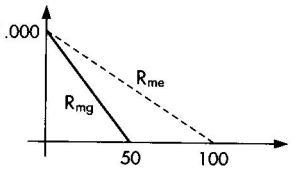


d) Receita marginal:

$$R_{mg}(x) = -20x + 1.000$$

Receita média:

$$R_{me}(x) = -10x + 1.000$$



31. a) 0,7 b) 0,3

32. a) $0{,}2y^{-0{,}5}$ b) 0,025 c) 0,975

33. a) $0{,}3y^{-0{,}5}$ b) 0,0375 c) 0,9625

34. a) 25/8 b) 25/9

35. a) 100 b) 50

36. a) $250\,L^{-1/2} - 6$ b) 244; 119; 77,3; 44; 19

37. 1

38. 3/7

39. 7/150

40. 3/58

41. 2

42. 3/11

43. a) $x < 200/3$ b) $\frac{200}{3} < x < 100$

44. a) $\frac{2 \cdot (50 - 0{,}5q)}{q}$
b) 4; 1,5; 0,67; 0,25 e 0.
c) infinito

45. a) 0,0909 b) 0,45%

46. a) 0,6% b) 1,2% c) 3%

47. 2.048

48. elasticidade igual a α

49. Demonstração.

50. Demonstração.

51. a) $f'''(x) = 36$
b) $f'''(x) = e^x$
c) $f'''(x) = -e^{-x}$
d) $f'''(x) = -\cos x$
e) $f'''(x) = \frac{2}{x^3}$
f) $f'''(x) = -\cos x + \operatorname{sen} x$
g) $f'''(x) = e^x - e^{-x}$

52. $\operatorname{sen} x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$

53. $\ln(1 + x) = x - \frac{x^2}{2!} + \frac{2!x^3}{3!} - \frac{3!x^4}{4!} + \ldots$

54. 0,4167

55. $\ln x = (x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \ldots$

56. $\cos x = \frac{1}{2} - \left(x - \frac{\pi}{3}\right)\frac{\sqrt{3}}{2} - \left(x - \frac{\pi}{3}\right)^2 \cdot \frac{1}{4} + \ldots$

57. a) 0 e) $\frac{1}{3}$
b) 0 f) 1
c) 0
d) 0

Capítulo 6

1. a) Crescente em R
b) Decrescente em R

c) $\begin{cases} \text{Crescente em: }]3/2; \infty[\\ \text{Decrescente em: }]-\infty, 3/2[\\ \text{Ponto de mínimo: } x = 3/2 \end{cases}$

d) $\begin{cases} \text{Crescente em: }]-\infty, 0[\\ \text{Decrescente em: }]0, \infty[\\ \text{Ponto de máximo: } x = 0 \end{cases}$

e) $\begin{cases} \text{Crescente em: }]2, \infty[\\ \text{Decrescente em: }]-\infty, 2[\\ \text{Ponto de mínimo: } x = 2 \end{cases}$

f) $\begin{cases} \text{Crescente em: }]-\infty, 3[\text{ ou }]4, \infty[\\ \text{Decrescente em: }]3, 4[\\ \text{Ponto de máximo: } x = 3 \\ \text{Ponto de mínimo: } x = 4 \end{cases}$

g) $\begin{cases} \text{Crescente em: }]-\infty, 1[\text{ ou }]2, \infty[\\ \text{Decrescente em: }]1, 2[\\ \text{Ponto de máximo: } x = 1 \\ \text{Ponto de mínimo: } x = 2 \end{cases}$

h) $\begin{cases} \text{Crescente em: }]-2, 2[\\ \text{Decrescente em: }]-\infty, -2[\\ \qquad\qquad \text{ou }]2, \infty[\\ \text{Ponto de máximo: } x = 2 \\ \text{Ponto de mínimo: } x = -2 \end{cases}$

i) Crescente em: $]0, 8[$
Decrescente em: $]-\infty, 0[$ ou $]8, \infty[$
Ponto de máximo: $x = 8$
Ponto de mínimo: $x = 0$

j) Crescente em R

k) Decrescente em R

l) Crescente em: $]0, \infty[$
Decrescente em: $]-\infty, 0[$
Ponto de mínimo: $x = 0$

m) Crescente em: $]-1, 0[$ ou $]1, \infty[$
Decrescente em: $]-\infty, -1[$ ou $]0, 1[$
Ponto de mínimo: $x = -1$ ou $x = 1$
Ponto de máximo: $x = 0$

n) Decrescente em: $]-\infty, 0[$ ou $]0, \infty[$

o) Decrescente em: $]-\infty, 2[$ ou $]2, \infty[$

p) Decrescente em: $]-\infty, 3[$ ou $]3, \infty[$

q) Crescente em: $]-\infty, 0[$
Decrescente em: $]0, \infty[$
Ponto de máximo: $x = 0$

r) Crescente em: $]-1, 1[$
Decrescente em: $]-\infty, -1[$ ou $]1, \infty[$
Ponto de mínimo: $x = -1$
Ponto de máximo: $x = 1$

s) Crescente em: $\left]-\infty, 2 - \frac{\sqrt{3}}{3}\right[$ ou $\left]2 + \frac{\sqrt{3}}{3}, \infty\right[$
Decrescente em: $\left]2 - \frac{\sqrt{3}}{3}, 2 + \frac{\sqrt{3}}{3}\right[$
Ponto de máximo: $x = 2 - \frac{\sqrt{3}}{3}$
Ponto de mínimo: $x = 2 + \frac{\sqrt{3}}{3}$

2. $x = 5/2$

3. $p = 20$

4. $p = 21$

5. $x = 4{,}65$ aproximadamente

6. $p = 8$

7. a) 52 b) 48,5

8. Máximo relativo: $x = 5$
Mínimos relativos: $x = 0$ e $x = 6$
Máximo absoluto: $x = 5$
Mínimo absoluto: $x = 0$

9. Máximo relativo: $x = 3$
Mínimos relativos: $x = 0$ e $x = 4$
Máximo absoluto: não existe
Mínimo absoluto: $x = 0$

10. Demonstração.

11. $C_{mg} = x^2 - 12x + 60$

12. Demonstração.

13. a) 12,16 b) Demonstração.

14. a) $C_{me}(x) = 40 - 10x + x^2$
b) Decrescente para $x < 5$ e crescente para $x > 5$; $x = 5$ é ponto de mínimo.

15. a) $C_{me}(x) = \frac{1}{3}x^2 - 4x + 30$
b) Decrescente para $x < 6$ e crescente para $x > 6$; $x = 6$ é ponto de mínimo.

16. Demonstração.

17. a) Demonstração. b) \$ 5,67

18. a) $C_{me}(x) = 0{,}1x + 3 + \frac{4.000}{x}$
b) $x = 200$
c) $x = 180$
d) $x = 200$

19. \$ 860

20. $x = 2{,}63$

21. $x = 141{,}42$

22. a) $C_{me} = x^2 - 20x + 400$ e $C_{mg} = 3x^2 - 40x + 400$
b) Demonstração.

23. \$ 38,63

24. a) $p = -0{,}005x + 20$ b) $p = 10$

25. $p = 7{,}36$

26. Demonstração.

27. a) 49,6 mil habitantes
b) Demonstração.
c) 50 mil habitantes

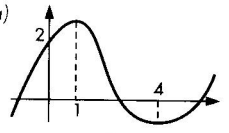
28. Demonstração.

29. 3 unidades de A e 6 de B.

30. $i = 12\%$ ao ano

31. a) 72,34% em A e 27,66% em B
    b) 9,94%

32. a) Côncava para cima em: R.
    b) Côncava para baixo em: R.
    c) Côncava para cima: $]3, \infty[$
       Côncava para baixo em: $]-\infty, 3[$
       Ponto de inflexão: $x = 3$
    d) Côncava para cima em: $]-\infty, 4[$
       Côncava para baixo em: $]4, \infty[$
       Ponto de inflexão: $x = 4$
    e) Côncava para cima em: $]-\infty, -8/3[$
       Côncava para baixo em: $]-8/3, \infty[$
       Ponto de inflexão: $x = -8/3$
    f) Côncava para cima em: $]-\infty, 1[$ ou $]3, \infty[$
       Côncava para baixo em: $]1, 3[$
       Pontos de inflexão: $x = 1$ ou $x = 3$
    g) Côncava para cima em: $]0, \infty[$
       Côncava para baixo em: $]-\infty, 0[$
    h) Côncava para cima em: $]-\infty, -1[$ ou $]1, \infty[$
       Côncava para baixo em: $]-1, 1[$
       Pontos de inflexão: $x = -1$ ou $x = 1$
    i) Côncava para cima em: $]1, \infty[$
       Côncava para baixo em: $]-\infty, 1[$

33. a)


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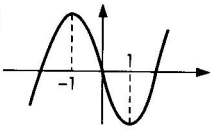

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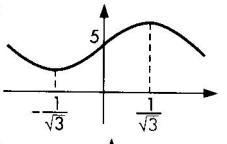

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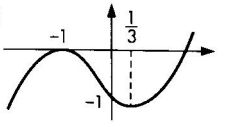

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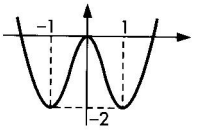

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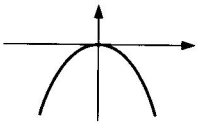

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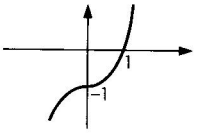

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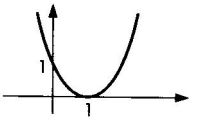

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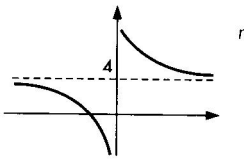

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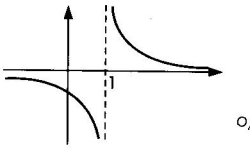

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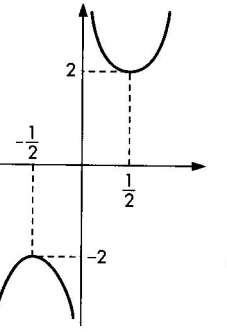
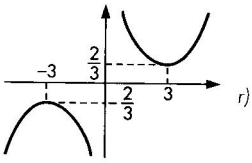
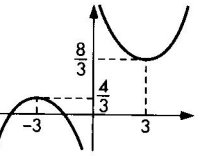
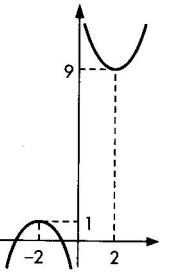
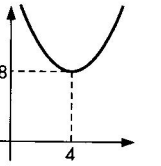
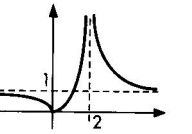


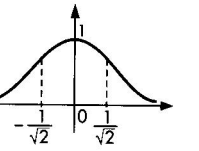
l)


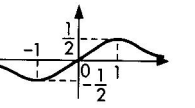
m)


n)


o)


p)


q)


r)


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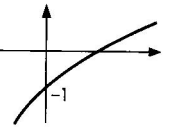

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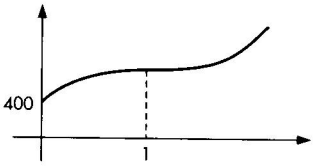
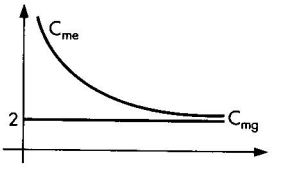
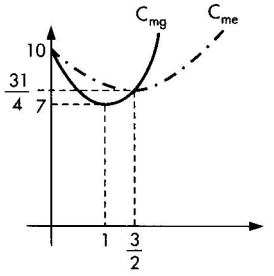
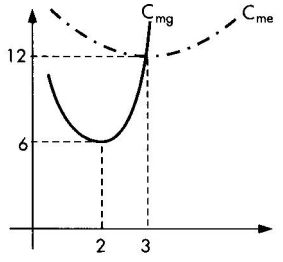


35. a) $C_{mg}(x) = 2$
    b) $C_{me}(x) = 2 + \dfrac{100}{x}$
    c)


36 a) $C_{mg}(x) = 3x^2 - 6x + 10$
b) $C_{me}(x) = x^2 - 3x + 10$
c)



37 a) $C_{mg}(x) = 6x^2 - 24x + 30$
b) $C_{me}(x) = 2x^2 - 12x + 30$
c)



38 a) Ponto de mínimo: $x = 2$
b) Ponto de máximo: $x = 3$
c) $\begin{cases} \text{Ponto de máximo: } x = 1 \\ \text{Ponto de mínimo: } x = 6 \end{cases}$
d) $\begin{cases} \text{Ponto de mínimo: } x = -2 \\ \text{Ponto de máximo: } x = 2 \end{cases}$
e) $\begin{cases} \text{Ponto de máximo: } x = -1 \\ \text{Ponto de mínimo: } x = 1 \end{cases}$
f) Ponto de mínimo: $x = -\frac{4}{3}$

39 30 m por 30 m

40 50 e 50

41 $r = \frac{10}{\sqrt[3]{4\pi}}$ e $h = \frac{500}{\pi r^2}$

42 25 m por 25 m

43 1

44 5 m por 10 m

45 5,66 m por 2,83 m

46 Demonstração.

47 base e altura iguais a 2,15 m ($\sqrt[3]{10}$)

48 2,71 m por 1,36 m

49 A base deve ser igual a 48 cm.

50 a) 13 b) Demonstração.

51 a) $x = 4{,}38$ b) Demonstração.

52 a) $x = 3$ b) Demonstração.

53 Demonstração.

54 33

55 $x = \frac{p - b}{2a}$

56 $x = \frac{\alpha - b}{2a + 2\beta}$

57 $x = 50/3$

58 $x = 47$

59 $x = 100$

60 $x = 75$

61 $x = 25$

62 a) \$ 70,00 b) \$ 90,00

63 a) \$ 510,00 b) \$ 511,00

64 a) $x = 16$ b) Demonstração.

65 32

66 10 garrafas

67 a) $R = 30x - 5x \ln x$ c) Demonstração.
b) $x = e^5$

68 2,63

69 \$ 860,00

70 a) \$ 300,00 b) 234,17

71 Demonstração.

72 Demonstração.

73 Exercício resolvido.

74.
a) 2.000 $\begin{cases} \text{manter: \$ 1.000,00} \\ \text{pedir: \$ 4.000,00} \\ \text{total: \$ 5.000,00} \end{cases}$

6.000 $\begin{cases} \text{manter: \$ 3.000,00} \\ \text{pedir: \$ 1.333,33} \\ \text{total: \$ 4.333,33} \end{cases}$

8.000 $\begin{cases} \text{manter: \$ 4.000,00} \\ \text{pedir: \$ 1.000,00} \\ \text{total: \$ 5.000,00} \end{cases}$

b) 4.000

75 Demonstração.

76 Demonstração.

Capítulo 7

1. a) $\frac{x^4}{2} + c$
   b) $\frac{x^3}{3} + \frac{3x^2}{2} + c$
   c) $\frac{x^3}{3} - \frac{3x^2}{2} + c$
   d) $5x - \frac{x^2}{2} + c$
   e) $5x + c$
   f) $\frac{3x^4}{4} - \frac{2x^3}{3} + 4x^2 - 6x + c$
   g) $5 \ln|x| + c$
   h) $\frac{x^3}{3} + 6 \ln|x| + c$
   i) $-\cos x + \operatorname{sen} x + c$
   j) $-\frac{1}{2x^2} + \frac{x^3}{3} - \frac{5x^2}{2} + c$
   k) $\frac{2x^{3/2}}{3} + c$
   l) $\frac{15x^{4/3}}{4} + c$
   m) $\frac{2x^{3/2}}{3} + \frac{3x^{4/3}}{4} + c$
   n) $x - 3 \ln|x| - \frac{5}{x} + c$
   o) $\operatorname{arc\,tg} x + \frac{x^3}{3} + c$
   p) $2e^x + c$
   q) $3e^x + \frac{x^4}{4} + c$
   r) $-\cos x - 5e^x + c$
2. Demonstração.
3. Demonstração.
4. Demonstração.
5. Exercício resolvido.
6. $C(x) = 0{,}05x^2 + 5x + 500$
7. $C(x) = 2x + 200$
8. a) $C(x) = 2x^3 - 3x^2 + 20x + 400$ b) 135
9. a) $C(x) = \frac{4}{3}x^3 - 3x^2 + 30x + 400$ b) $\frac{385}{3}$
10. $R(x) = 50x - \frac{x^2}{2}$
11. a) $R(x) = 20x - x^2$ b) $R_{me}(x) = 20 - x$
12. a) $R(x) = 100x$ b) $R_{me}(x) = 100$
13. a) $L(x) = 3x - 100$ b) 100/3
14. $x = 4$
15. $C(x) = 0{,}04x^2 + 4x + 26$
16. $P(x) = -x^2 + 40x$
17. $P(x) = 20 \cdot x^{0.5}$
18. $C = 0{,}8y + 100$
19. $S = 0{,}2y - 100$
20. a) $C = y^{0.5} + 50$ c) $1 - \frac{1}{2}y^{-1/2}$
    b) $S = y - y^{0.5} - 50$
21. a) 15 g) 4,5
    b) 12 h) 20/3
    c) −22,5 i) 3/2
    d) 8/3 j) $e^3 - 1$
    e) $\frac{44}{3}$ k) 0
    f) $\frac{-125}{6}$ l) 0
22. a) $\frac{1}{3}$ h) $\frac{8}{3}$
    b) 9 i) 4
    c) ln 2 j) 22/3
    d) 4 ln 2 k) 1/6
    e) 8/3 l) 1/6
    f) 4,5 m) 8/3
    g) 4 n) 8/3
23. a) 1/4 b) 9/2 c) 1/3
24. Exercício resolvido.
25. a) 1/3 b) 1 c) e^1
26. \$ 342.000.000,00
27. \$ 113.569.219,00
28. \$ 243.456.000,00

29. a) $x = 5$ e $p = 10$ b) 25 c) 12,5

30. a) $x = 2$ e $p = 19$ b) 2 c) 16/3

31. a) $x = 10$ e $p = 20$ b) 50 c) 400/3

32. a) $\frac{1}{3}\ln|4 + 3x| + c$

b) $-\ln|5 - x| + c$

c) $\ln|\ln x| + c$

d) $\frac{1}{2}e^{2x} + c$

e) $\frac{1}{2}e^{2x+3} + c$

f) $e^{\operatorname{sen} x} + c$

g) $\frac{2}{3}(x^3 + 1)^{1/2} + c$

h) $\frac{\operatorname{sen}^2 x}{2} + c$

i) $\frac{2}{3}(1 + \ln x)^{3/2} + c$

j) $\frac{(x^2 + 3)^5}{5} + c$

k) $\frac{1}{24}(3x^2 + 1)^4 + c$

l) $\ln(2x^2 + 3) + c$

33. $V = 20\ln(5 + x) - 13$

34. a) $\frac{1}{8} - \frac{1}{2(1 + e)^2}$ b) $\frac{1}{3} \cdot 2^{3/2} - \frac{1}{3}$

35. a) $\frac{x^2}{2}\ln x - \frac{1}{4} \cdot x^2 + c$

b) $-x \cdot \cos x + \operatorname{sen} x + c$

c) $x \operatorname{sen} x + \cos x + c$

d) $-e^{-x} \cdot (x + 1)$

e) $e^x(x^2 - 2x + 2) + c$

f) $\frac{x^{n+1}}{n+1}\left(\ln x - \frac{1}{n+1}\right) + c$

g) $-\frac{\operatorname{sen} x \cos x}{2} + \frac{x}{2} + c$

36. a) $\ln\left|\frac{(x-2)^3}{x-1}\right| + c$

b) $\ln\frac{|x|}{(x^2 + 1)^{1/2}} + c$

c) $\frac{x^3}{3} + \frac{1}{3}\ln|x^3 - 1| + c$

d) $\frac{x^2}{2} - 2x + \frac{1}{6}\ln\left|\frac{x-1}{(x+1)^3}\right| + \frac{16}{3}\ln|x + 2| + c$

37. Demonstração.

38. Demonstração.

39. Demonstração.

40. Demonstração.

41. Demonstração.

42. $y = 10 \cdot e^{-8x}$

43. $y = x^3 - 2x^2 - x + 9$

44. $y = \ln x + 18$

45. $y = \frac{5}{2}x^2$

46. a) $y = \frac{5}{2}x^2 + c$

b) $y = \frac{-1}{\frac{3}{5}x^5 + c}$

c) $y = \sqrt[3]{3e^x + c}$

d) $y = c \cdot e^x - 8$ ou $y = -c \cdot e^x - 8$

e) $y = 50 - c \cdot e^{-10x}$ ou $y = 50 + c \cdot e^{-10x}$

47. $y = 80 - 80 \cdot e^{-0,5x}$

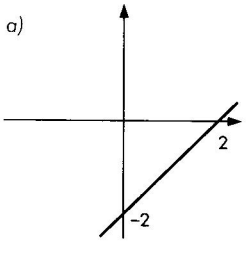
48. $y = 3 + 2e^{\frac{x^2-1}{2}}$

49. $y = \sqrt[4]{2x^2 + 9.992}$

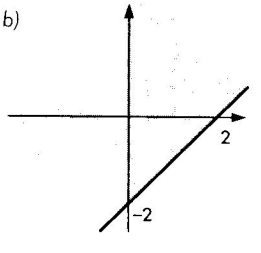
50. $y = \frac{15e^x}{15e^x - 14}$

Capítulo 8

1. a)



b)



c)