



Yuri Ki

Heterodimensional cycles of co-index two and symbolic blenders

Tese de Doutorado

Thesis presented to the Programa de Pós-Graduação em Matemática of the Departamento de Matemática, PUC–Rio, as partial fulfillment of the requirements for the degree of Doutor em Matemática.

Advisor: Prof. Lorenzo J. Díaz

Rio de Janeiro
November 2012



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Rio de Janeiro, 09/11/2012

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Bibliographic data

Ki, Yuri

Heterodimensional cycles of co-index two and symbolic blenders / Yuri Ki; advisor: Lorenzo J. Díaz . — 2012.

118 f. : il. ; 30 cm

Tese (doutorado) - Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Matemática, 2012.

Inclui bibliografia

1. Matemática – Teses. 2. Aplicação produto cruzado. ; 3. Atrator de Hutchinson. ; 4. Blender. ; 5. Blender simbólico. ; 6. Ciclo heterodimensional. ; 7. Ciclo robusto. ; 8. Intersecção homoclínica forte. ; 9. Hiperbolicidade parcial. ; 10. Sistemas de funções iteradas. . I. Ki, Yuri. II. Pontifícia Universidade Católica do Rio de Janeiro. Departamento de Matemática. III. Título.

CDD: 510

Acknowledgments

I would like to thank CNPq and PUC-Rio for the financial support.

I would like to express my sincere gratitude to my advisor Lorenzo J. Díaz for his advising, availability and patience.

To the jury members: Alex Lúcio Ribeiro Castro (PUC-Rio), Enrique Ramiro Pujals (IMPA), Isabel Lugao Rios (UFF), Jairo da Silva Bochi (PUC-Rio), Paul Alexander Schweitzer (PUC-Rio), Rafael Oswaldo Ruggiero Rodriguez (PUC-Rio) and Vilton Jeovan Viana Pinheiro (UFBA).

I also would like to thank to my co-workers Artem Raibekas and Pablo Barrientos.

Thanks to Christian Bonatti, Enrique Pujals, Rafael Potrie and Thiago Catalan for helpful and useful mathematical conversations.

There are a lot of people of Departamento de Matemática i would like to thank, in particular, Aninha, Creuza, Kátia, Monica, Nicolau, Renata, Sinésio. I would like to thank Flávio Abdenur for carrying the “dynamical team” during these years at PUC. I would like to thank Thomas Lewiner for helping with latex.

Thanks to all friends from PUC-Rio, in particular, Felipe, Miguel and *a velha-guarda* for all conversations in our favorite place at the departament, the kitchen!!

Obviously, i would like to thank the *caos*-girls: Cris, Débora, Emília, Jaque, Lis and Paty for all jokes, laughs, laughs and more laughs!

Thanks to the *amigas-band*, for being present in all important moments of my life!

My deep gratitude to my parents om-ma and ap-pa, and my sister Hanna. I always have with them the support and unconditional love.

A very special acknowledgment to my family: Pablo Guarino for helping me, taking care of me and making me laugh every day and all the time! Thanks for being part of my life and making me the happiest person of the world!

Abstract

Ki, Yuri; Díaz, Lorenzo J.. **Heterodimensional cycles of co-index two and symbolic blenders**. Rio de Janeiro, 2012. 118p. Tese de Doutorado — Departamento de Matemática, Pontifícia Universidade Católica do Rio de Janeiro.

In the first part of the thesis, we consider diffeomorphisms having heterodimensional cycles associated with a pair of saddles P and Q of co-index two. We prove that diffeomorphisms with cycles, which have at least one pair of non-real central eigenvalues, generate robust heterodimensional cycles. Moreover, when both central eigenvalues are non-real, the robust cycles are associated with the continuation of the initial saddles (i.e. the cycle can be *stabilized*). In the second part of this work we study skew product maps over Bernoulli shifts with contracting fibers and Hölder dependence on the base points. We prove that systems satisfying the covering property exhibit symbolic blenders. These blenders are generalizations of the usual blenders whose main property is that their central direction may have any dimension $d \geq 1$.

Keywords

Blender; Heterodimensional cycle; Hutchinson attractor; Iterated function system; Partial hyperbolicity; Robust cycle; Skew product maps; Strong homoclinic intersection; Symbolic blender.

Resumo

Ki, Yuri; Díaz, Lorenzo J.. **Ciclos heterodimensionais de co-índice dois e blenders simbólicos**. Rio de Janeiro, 2012. 118p. Tese de Doutorado — Departamento de Matemática, Pontifícia Universidade Católica do Rio de Janeiro.

Na primeira parte da tese, consideramos difeomorfismos com ciclos heterodimensionais associados a um par de selas P e Q de co-índice dois. Provamos que difeomorfismos com ciclos que possuem no mínimo um par de autovalores centrais do ciclo não real geram ciclos heterodimensionais robustos. Além disso, quando os autovalores centrais são não-reais, os ciclos robustos estão associados as continuções das selas iniciais (ou seja, os ciclos podem ser *estabilizados*). Na segunda parte deste trabalho estudamos mapas produto cruzado sobre aplicações deslocamento (do tipo Bernoulli) com fibras contrativas e dependência Hölder nos pontos da base. Provamos que sistemas que satisfazem a propriedade de cobertura exibem blender simbólicos. Estes blenders são generalizações do blender usual cuja principal característica é que suas direções centrais podem ter qualquer dimensão $d \geq 1$.

Palavras-chave

Aplicação produto cruzado; Atrator de Hutchinson; Blender; Blender simbólico; Ciclo heterodimensional; Ciclo robusto; Intersecção homoclínica forte; Hiperbolicidade parcial; Sistemas de funções iteradas.

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