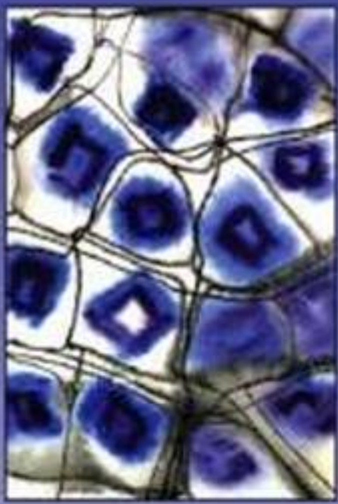
URBAN OPERATIONS RESEARCH



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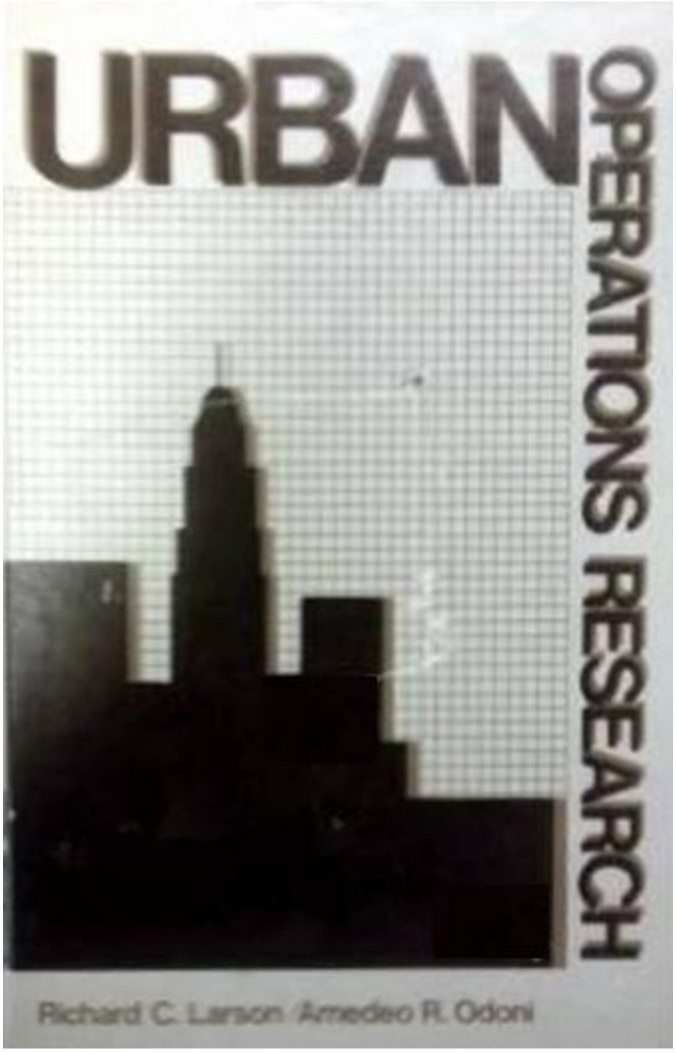

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Introduction

Approximately 68 percent of the citizens of the world's developed countries live today in cities or in city-centered metropolitan areas.[1] The economic and social fabric of these high-density clusters is elaborately interwoven, with the well-being of each citizen intricately enmeshed with the activities of others. Strong interdependencies arise in all areas of human need: food, shelter, safety, clothing, recreation, maintenance, energy provision, and so on. Servicing these needs requires highly structured transportation and communication networks throughout the city for effective provision of a variety of *urban services*: emergency medical, police, mail collection and delivery, fire protection, street and highway maintenance, utility repair, snowplowing, street cleaning, refuse collection, bus and subway transportation, taxi transportation, and so on.

Increasingly, citizens are demanding more urban services, by type, quantity, and quality. Yet the ability of most cities in the United States and elsewhere to pay for additional services has been severely strained during the 1970s. The resulting pressure, between the demands for more and better services, on the one hand, and decreased costs, on the other, has created a strong need for improved management decision making in urban services. It is a primary purpose of this book to provide methods for assisting these decisions.

[1] *United Nations Demographic Yearbook 1976.* United Nations Department of Economic and Social Affairs Statistical Office, New York, 1977 (28th issue), Table 6, pp. 146–165.

For our purposes, a decision is an irrevocable allocation of resources.[2] Thus, this book will deal with the allocation or deployment of the resources of urban service systems, including personnel, equipment, and various service-improving technologies. From this viewpoint, urban operations research can be thought of as a decision-aiding technology, one to assist urban managers in improving the deployment of their resources. Most deployments occur spatially throughout the city, so much of our work will have a strong spatial component.

Urban operations research is not new. In 1736, the famous mathematician Leonhard Euler was confronted with an urban deployment problem when he attempted to find a feasible parade route over the seven bridges of Königsberg (now Kaliningrad) such that no bridge was crossed more than once. As argued in Chapter 6, Euler found the assignment impossible, but as a by-product founded the extremely useful field of graph or network theory. Much more recently (1937), Merrill Flood of Columbia University is credited by George Dantzig and others as having stimulated serious interest in the "traveling salesman problem" (see Chapter 6) through his efforts to route school buses more efficiently. In the 1950s, Leslie Edie and his group at the New York Port Authority applied operations research methods to improve management of New York's tunnels and bridges, an activity for which Edie was awarded the first annual Lanchester Prize by the Operations Research Society of America (ORSA) for the best published work in operations research. Very significant progress in urban operations research has occurred within the last decade, spurred by the New York City Rand Institute and by research at MIT, the State University of New York at Stony Brook, Carnegie–Mellon University, the University of Maryland, Columbia University, and several other universities and organizations. Two of these efforts were awarded Lanchester Prizes by ORSA. Much of this recent work is made accessible herein, both in the text and in the end-of-chapter problems.

In the remainder of this chapter we provide a motivation for the material in Chapters 2–8. Costs of providing services are always important, so in Section 1.1 we discuss the recent tendency for the costs of urban services to grow at rates faster than those of many other sectors of the economy. In Section 1.2 we present thumbnail sketches of situations in which urban operations research methods are required to help analyze various urban deployment problems. In Section 1.3 we discuss briefly the steps that are

[2] ". . . irrevocable in the sense that it is impossible or extremely costly to change back to the situation that existed before making the decision. Thus for our purposes a decision is not a mental commitment to follow a course of action but rather the actual pursuit of the course of action." R. A. Howard, "Decision Analysis: Applied Decision Theory," in *Proceedings of the Fourth International Conference on Operational Research*, D. B. Hertz and J. Melese, eds., Wiley, New York, 1966, pp. 55–71.

necessary in undertaking any operations research study. Finally, in Section 1.4 we argue for the necessity of a probabilistic rather than deterministic approach to most of the problems we will be addressing.

1.1 COST EFFECTIVENESS OF URBAN OPERATIONS RESEARCH STUDIES

The potential benefits of an urban operations research study can be appreciated better once one becomes aware of both the magnitude of current expenditures for urban services and the rate at which these expenditures have been increasing in recent years. Table 1-1 shows the cost of various services for 35 of the largest cities in the United States (including the 15 largest ones) for the years 1959 and 1973 [ODON 77].[3] After deflating the 1973 figures to 1959 prices (the total nationwide inflation between 1959 and 1973 was 51 percent), the last column of Table 1-1 presents the percentage increases [in *constant* (1959) dollars]. The increase in per capita gross national product (GNP) during the same period (in 1959 dollars) amounted to 49 percent, or one-third of the percentage increase in total city expenditures. This growth in total expenditures cannot be attributed to large population increases, since the total population of the 35 cities covered by Table 1-1 increased by less than 2 percent between 1959 and 1973. As seen from the table, only expenses for highways lagged behind the rate of growth of the national economy (as reflected in GNP per capita), while expenses for sanitation departments just kept pace with that growth. Costs of most other urban services over the past generation have increased two, three, or more times as rapidly as have costs in other sectors of the economy.

The absolute expenditures are impressive in themselves. New York City now spends more than $2 billion a year for police, fire protection, and sanitation alone. This amount exceeds the annual budgets of many national governments in the world. It is also interesting that one of the fastest-growing items in Table 1-1 is the interest that city governments pay annually to service their debts.

Probably more pertinent to the type of issues that we shall be addressing are unit costs, such as average salaries. The average salary (with no overtime pay) for a policeman or a fireman in large U.S. cities in 1979 was about $19,000. In addition, fringe benefits (such as health insurance, retirement plans, etc.), and overtime pay cost cities another 40 percent (approximately) of that amount [ODON 77]. Thus, the average cost per rank-and-file police department or fire department employee to the city was about $27,000 per year.

[3] Odoni, A. R., "Recent Employment and Expenditure Trends in City Police Departments in the United States," *Journal of Criminal Justice*, *5*, 119–147 (1977).

TABLE 1-1 Total costs of selected U.S. urban services computed for 35 of the largest U.S. cities (including the 15 largest in 1973) for 1959 and 1973.

Cost Item	*1959** ($000)	*1973* ($000)	*1973 (deflated*)* ($000)	*Increase in Constant Prices* (%)
Police	569,117	1,886,343	1,249,233	120
Fire	326,693	895,697	593,176	82
Sanitation	256,988	576,246	381,619	48
Parks	238,832	591,285	391,579	64
Hospitals	360,952	1,438,440	952,609	164
Highways	520,247	755,873	500,578	−4
Public welfare	447,117	3,151,638	2,087,177	367
Education	832,088	3,511,335	2,325,387	179
Interest on general debt	199,080	860,488	569,859	186
Total expenditures	5,203,329	19,564,637	12,956,713	149
GNP per capita	$2,720	$6,122	$4,054	49

*1959 dollars.

It is instructive to consider the cost of, say, a two-officer police patrol car, operating 24 hours per day throughout the year. With 8,760 hours in a year and with police officers working an average of about 1,850 hours per year in large cities [ODON 77], operation of the patrol car requires approximately 10 officers (assuming that personnel scheduling is competently performed), corresponding to $270,000 per year! The fact that the patrol car itself, the equipment it includes, and the fuel it consumes probably cost less than $10,000 per year also serves to illustrate how labor-intensive the police patrol car service is. Indeed, it has been found that in large police departments in the United States, labor-related costs have consistently amounted to about 90 percent of annual department costs over the past 20 years [ODON 77]. To make costs even higher, some police and fire departments now have two approximately equal-sized labor pools: one active and one retired, the latter costing about as much per year as the former. These facts immediately present the urban operations research analyst with a challenge. Small gains in productivity (even of the order of 1 to 2 percent) can mean large savings to a city in monetary terms. Analyses such as those that will be described in this book have the potential for being very cost-effective.

There are two very simple ways in which we can evaluate the *cost effectiveness* of a study. The first applies when the study results in a net decrease in costs, assuming that service quality remains unchanged. Then the *benefit/cost ratio* of the study is the ratio of the present discounted value of savings achieved by implementing the study to the cost of the study. For instance, if a study costing \$50,000 results in a cost savings of \$25,000 per year—projected indefinitely—then at a discount rate of 8 percent, the present value of the savings achieved is

$$\frac{25{,}000}{1.08} + \frac{25{,}000}{(1.08)^2} + \dots + \frac{25{,}000}{(1.08)^t} + \dots = 25{,}000 \sum_{t=1}^{\infty} \left(\frac{1}{1.08}\right)^t$$
$$= 25{,}000 \frac{1}{0.08} = \$312{,}500$$

The benefit/cost ratio of such a study would then be \$312,500 : \$50,000 or 6.25 : 1.0. Such a study is quite cost-effective.

The second method for evaluating the cost effectiveness of a study applies when there is no net decrease in costs, but service quality is improved. This is more difficult to carry out in practice because of the problems involved in measuring service quality for most urban services. Still, suppose that according to several accepted measures of service quality, we can make a statement such as the following: "Implementing the results of the study improved service quality by an amount that would have required N additional personnel under the former operating procedures." Then the present discounted cost of the N additional personnel, who are now not needed, can be used to compute the benefit/cost ratio of the study.

What is surprising about most such computations is the small percentage in service improvement or cost reduction that is needed to make most studies cost-effective. As argued in Chapter 8, this line of thinking is not currently popular in cities because rarely can one decrease costs outright, and performance measures to gauge service quality are often found difficult to state explicitly. Still, any reasonable approximate measure of service quality improvement will often yield a value of 5 or more as the anticipated benefit/cost ratio of a study.

1.2 PROBLEMS WELL SUITED TO URBAN OPERATIONS RESEARCH

The following letter appeared in the *Washington Post* on Sunday, January 28, 1979:

> *Bunched-Up Buses*
> I have long been trying to discover why Metro buses on Wisconsin Avenue are regularly bunched during rush hours instead of being evenly spaced. It is a common experience to wait for 10 or 15 minutes in rain or snow, and then find three or four buses coming along together nose to tail.

As Metro refuses to reply to letters on this subject, I can only assume that it has something to hide.

From the passenger's point of view, the advantages of even spacing are obvious—shorter waits and buses that are evenly filled, instead of being packed up at the front and half-empty behind. Traffic congestion is also eased.

Evidently, either the company or its drivers must prefer bunching. I continue to wonder why.

E. PETER WRIGHT

This letter is typical of similar letters on this subject that invariably appear several times each year in our major newspapers. The phenomenon is often referred to as the "clumping" or "bunching" of urban buses and is among the most irritating in the daily experiences of many of us. If the key performance measure of a bus transit system is curbside wait, then clumped buses can increase mean curbside wait by a factor of 2 or more compared to a system with evenly spaced buses. In effect, a system of buses clumped in pairs can reduce a system of N buses, each having capacity C, to a system with $N/2$ "macro buses," each having capacity $2C$. It is apparent that a study to decrease clumping and hence curbside waits can, in effect, substitute for the number of additional buses that would have been necessary under old operating procedures to achieve the same curbside-wait reduction. Many such studies can be extremely cost-effective according to the curbside-wait measure.

Dissatisfaction with urban service provision is not limited to buses. In the late 1960s, citizens complained in letters to the editor of the *New York Times* of incurring delays in excess of 20 minutes prior to the answering of New York's (then new) 911 emergency telephone number. Other complaints span the spectrum of urban services, from negligence in pothole repair, to late arrival of emergency medical services, to slow removal of snow from the streets. In the remainder of this section we provide thumbnail sketches of deployment problems that are particularly well suited to an urban operations research approach. However, we do not wish to deceive the reader with the apparent simplicity of these problems; implementation of any proposed approach is a difficult process, requiring sensitivity to many of the issues raised in Chapter 8.

Example 1: Value of (Partial) Information on Vehicle Location

The Bonded Taxi Company has been losing revenue recently because of the overambitiousness of some of its own drivers. The radio dispatcher usually assigns that cab which claims to be closest to the scene of a call for cab service. However, some of the more ambitious drivers have been claiming to be "right around the block," when actually they might be at a distant location. Such a car takes a long time to reach the scene; upon arrival, the driver finds either a very dissatisfied customer who will never again use Bonded or no customer at all (because the customer called another cab company).

The Urbtronics Corporation has offered to sell Bonded a $\frac{1}{4}$-mile-resolution car-locator system, which, it is claimed, would provide the dispatcher with accurate position information, thereby eliminating distant dispatches caused by overly ambitious drivers. Other advantages of the car-locator system would include safety to drivers and better tracking of actual passenger mileage.

You have been hired to evaluate the advantages (and disadvantages) of the car-locator system. What do you do?

Example 2: Location of Garbage Incinerators

A mayor of a large city has discovered, to his surprise and consternation, that the city's refuse-collection trucks spend more time every day in moving back and forth between the city and a few remote garbage dumping sites that the city uses than in collecting garbage from city streets. It has been decided that several environmentally safe garbage incinerators will be built at a number of isolated locations within the city's boundaries, so that travel distances for unloading garbage can be reduced drastically. About 10 potential (and politically feasible) sites have been identified. The following interrelated questions must now be answered:

a. How many incinerators should be built?

b. Where, among the available sites, should the incinerators be located?

c. Which "truck runs" should be assigned to which incinerator?

Example 3: School-Bus Routing

For many suburban communities, especially those with relatively low population densities, the cost of transporting students to and from the community schools represents an important fraction of the annual school budget. In a large number of these cases the design of bus routes has been made on a haphazard basis with new bus stops and routes added as new children entered the school system. It is often possible to find instances where two or more different buses make stops at the same location to pick up different sets of students attending the same school. An additional problem is that, because of inefficient route design, some students are transported to school much before class time, thus increasing the need for the presence of supervisors at the schools, for additional recreational facilities, and so on.

An improved route-design process would offer the dual benefit of (1) reducing the number of bus runs and transportation costs, and (2) improving the quality of service to the students. Can such a process be devised?

Example 4: Relocation Strategies for Fire Companies

Although the *average* utilization of firemen is less than 25 percent in a particular city, the city is still experiencing unusually long delays in reaching some serious fires. These occur because demand patterns peak between 7:00 P.M. and 9:00 P.M., and one large fire (or several smaller ones, or false alarms) can "clean out" the fire stations in an area. The department has previously allowed dispatchers to *move up* (or relocate, reposition) certain companies. But with current levels of congestion they cannot adequately perform this task, because of heavy peak workloads and a lack of quantitative guidelines on how the relocation is to be done.

You are working for SOLVEIT Consulting Associates. You have been hired by the fire department to devise improved relocating strategies. A real-time computer capability is being implemented by the department and you have the freedom to use the computer in your system design. Design an approach and a solution method for this problem.

Example 5: Effects of a Law on Police Tours of Duty

Since the state legislature passed the "three-tour statute" in 1922, the police department of a large city has been constrained by law to allocate an equal number of police officers to each of the three tours of duty (midnight to 8:00 A.M., 8:00 A.M. to 4:00 P.M., and 4:00 P.M. to midnight). In recent years this constraint has been particularly troublesome, since near-saturation loads occur during predictable periods, but to relieve the congestion, additional officers would have to be added to the force *around the clock*. This is prohibited by budgetary considerations.

Examining this situation, the in-house planning and research group sees an opportunity to use simulation and analytical models in a very important way. Instead of accepting the existing statute as a "given" constraint, the group plans to examine how the patrol force would function *if* the law were modified to allow tours with nonequal numbers of operating personnel. The group is convinced that the current total number of officers available is sufficient to handle the needs of the city if the tours could be restructured (perhaps even allowing overlapping tours) to reflect the widely varying call-for-service rates and the needs for preventive patrol.

The group initially plans to use queueing models to get a rough idea of the number of personnel required by place and by time of day to achieve a "reasonable level of service." Then response and patrol models will be used to structure thinking about sector design, workloads, preventive patrol coverage, and so on. Finally, several detailed simulation tests will be performed to determine the extent of improvement obtained by reallocating the officers. If the results are sufficiently promising, the group plans to make the findings of the study publicly available. Eventually, it is hoped that this may cause a revision in the current law.

Working as the study group, how do you proceed?

Example 6: Redesigning a City's Ambulance Services

Currently, a city's emergency ambulance needs are handled by several private companies. These companies have been facing a deteriorating financial situation, with labor and maintenance costs increasing and a growing number of indigents unable to pay the cost of ambulance service. The companies cannot afford the expense of highly trained drivers and attendants; but a recently passed state law requires that all drivers have substantial paramedical training by January 1 of next year. Given this situation, it is highly probable that the private companies will go out of business and that some other means of providing ambulance service will have to be provided.

The mayor's office has requested that a study group examine and evaluate alternative proposals for providing ambulance service:

a. Incorporate ambulance service into police department operations.

b. Have a separate city-sponsored ambulance fleet.

c. Subsidize current companies or a merged version of those companies.

As the appointed study group, structure and analyze this problem, paying particular attention to operational, economic, and service-related issues.

1.3 STEPS IN AN OPERATIONS RESEARCH STUDY

In tackling problems such as those presented in Section 1.2, one must necessarily follow the eight classical steps of a *systematic analysis* which are summarized in Figure 1.1.

As an illustration of utilizing these steps, let us consider Example 6 in Section 1.2, "Redesigning a City's Ambulance Services." Here a law has been passed mandating upgraded emergency medical skills for ambulance drivers and attendants. Private ambulance companies that are currently providing service are likely to go out of business unless something is done. A study group has been appointed by the mayor to analyze the problem. Let us briefly consider each of the eight steps, as they might evolve in practice:

1. **Define the problem.** With private ambulance companies likely to go out of business, the problem as stated to the study group is to determine the economic, operational, and service-related ramifications of alternative proposals for the provision of emergency ambulance service. Broadened to include the mayor and his or her staff, the problem is to select and implement a (possibly) new form of ambulance service.

2. **Identify the objectives.** Often, objectives are stated broadly; for instance: " The objective is to deliver effective and efficient emergency

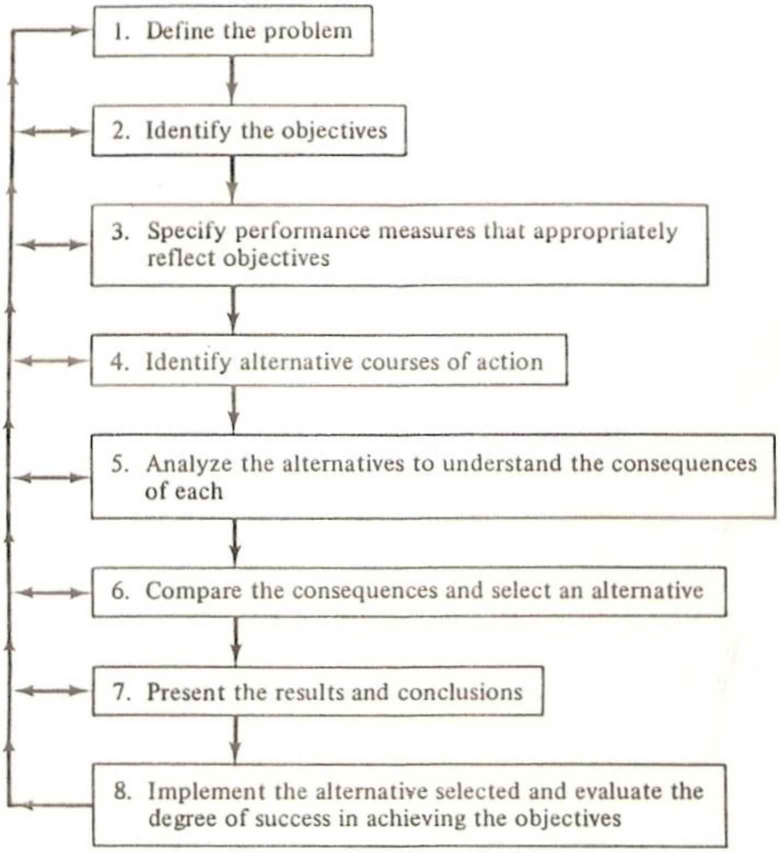

FIGURE 1.1 Eight steps in an operations research study.

ambulance service equitably to all citizens throughout the city, within given fiscal constraints."

3. **Specify performance measures.** To analyze the various alternatives, we must have a systematic procedure for evaluating how well each one accomplishes the objective(s) stated in Step 2. Performance measures allow us to undertake the quantitative part of our study. A performance measure is a quantity attributable to system operation; it reflects the quality of system performance, or, at least, it quantifies some aspect of system performance. A good performance measure is understandable to both citizens and agency personnel, is relatively stable under some set of operating conditions, is dependent on operating policy, is readily measurable, and is not easily subvertible (i.e., it is a true measure of system quality or at least service quantity). For some relatively straightforward urban services, such as door-to-door pickup and delivery services, valid performance measures are easy to identify. For others, however, one must rely on surrogate measures that represent an intermediate reading of service

quality; this is particularly true of services for which we do not now know "production functions" that relate service characteristics to achievement of fundamental objectives. Examples are the relationships (or lack of known relationships) among police and crime, transportation services and economic growth, and—in our case here—ambulance deployment and mortality and morbidity. Thus, we may have to settle for such surrogate measures as response times to various types of ambulance calls for various neighborhoods, workloads or utilization rates of the ambulances, costs of training and retaining personnel, and other system costs. (As argued in Chapter 8, there are many important nonquantifiable features of a systematic analysis that require consideration in addition to performance measures.)

4. **Identify the alternative courses of action.** Here the basic alternative courses of action are prespecified:

    a. Incorporate ambulance service into police department operations.

    b. Have a separate city-sponsored ambulance fleet.

    c. Subsidize the existing companies or a merged version of those companies.

    Of course, each of these can be implemented in numerous different ways, thus implying that there are, in fact, many alternatives that we should consider. A reasonable strategy would be to compare the best option under each of (a), (b), and (c) in order to select the best basic strategy.

5. **Analyze the alternatives to understand the consequences of each.** At this step the methods of Chapters 2–7 are most appropriate and necessary for examining the operational consequences of alternative courses of action. As one example, we could employ our analysis to examine the implications of "equal service provision" for each of the three basic options. Equal service could imply for each option the achievement of, say, a 5-minute average citywide ambulance response time, with no neighborhood incurring an average response time greater than 8 minutes. We would then construct mathematical models of each of the three systems proposed—one augmenting the current police patrol force, the second representing an independently operating ambulance fleet, and the third representing the current companies or a merged version of those companies—and analyze each of the models, while maintaining as a constraint the equal-service provision. The geometrical probability methods of Chapter 3

would probably be useful in depicting geographical interactions, and the spatial queueing models of Chapter 5 would be necessary for analysis of system congestion. One might also require the network methods of Chapter 6 to study the point-to-point transportation characteristics of the city, and perhaps even the simulation techniques of Chapter 7, especially if the proposed operations are too complicated to yield a mathematically tractable model.

6. **Compare the consequences and select an alternative.** This is simply an extension of Step 5. If service levels are held equal for each option, then selection will probably be based on cost and on nonquantifiable issues such as political feasibility, likely implementation time, and long-term impact of the decision on other services.

7. **Present the results and conclusions.** The study group must present its findings to the mayor and his or her staff. Usually, an oral presentation highlighting the main features of the analysis is more effective than a long formal report (although thorough written documentation is obviously necessary, as well). In the operations research profession, there is a school of thought which argues that the analysts should not select an option in Step 6 but should present the characteristics and consequences of each to the decision maker in Step 7, who, in turn, will make the selection. Another school of thought argues that the analysts probably know more about the problem than anyone else is likely to, and thus—provided that the agreed-upon performance measures are adequate—are in the best position to recommend an alternative. In many cases, a hybrid strategy seems most appropriate, in which a constructive dialogue is established between the analysts and the decision makers, and the selection is eventually made jointly.

8. **Implement and evaluate.** As argued in Chapter 8, this crucial step is often the most difficult. Especially in urban services, rarely are all the constraints and other relevant features of a problem visible before attempted implementation. One should anticipate a period of "shakedown," "debugging," or "fine tuning" during the initial stages of any implementation program. Information obtained during this phase will often "feed back" to one of the earlier seven steps in the analysis, making the entire analysis process iterative in nature. *Rarely is the analysis process strictly linear or sequential.* This is why we have indicated possible feedback paths in Figure 1.1. For instance, under any of the three basic ambulance options, once the spatial and temporal deployment of ambulances is made known, various neighborhood groups might raise objections because of perceived or actual undercoverage of their area by the proposed ambulance service.

If the complaints are found to be valid, the analysts may have to go back to Step 4 and redefine the alternative deployments that are feasible. Problems could also arise in labor–management negotiations, in purchasing equipment, in training, or in a host of other cases. Even if visible problems do not occur, it is necessary to monitor and evaluate the implementation process to assure that the system is functioning as planned and designed.

Much of the work in Steps 3–6 will involve working with *models*, idealized mathematical representations of the urban service systems of interest. A primary purpose of Chapters 2–7 is to develop skills in formulating such models. It is important to emphasize at this early point that the type and complexity of models must be adjusted to the types of questions that will be addressed during the eight steps outlined above. A given urban service system may need to be represented by two *entirely dissimilar* models in the process of answering two entirely dissimilar sets of questions about that system. In addition, just like the entire eight-step procedure, the formulation of a model only rarely turns out to be a "single-pass" sequential process. It is often necessary to revise a mathematical model several times in the course of an urban operations research study before that model is tailored to the particular needs of that study.

Many of the people who will be constructing models of urban service systems will also have responsibility for implementing them or the policy results of their use in one or more cities. Thus, although most of the methods of this book pertain to Steps 3–6, it is important to understand the larger framework in which a public-sector analysis is carried out. Our brief treatment here is sufficient at this time to provide the perspective necessary for Chapters 2–7. A fuller treatment is given in Chapter 8.

1.4 NEED FOR A PROBABILISTIC ANALYSIS

Most urban services face uncertainties related to time of occurrence, type, location, and quantity of demands. Considering an individual household, for instance, the amount of refuse it generates, the amount of mail it sends and receives, its need for emergency assistance (from police, fire, ambulance, or emergency repair services), and its use of public and private transit facilities fluctuate from day to day. True, there are regular statistical patterns that allow us to predict reliably the percentage of transit riders that will use a system between, say, 5:00 and 6:00 P.M., or the fraction of a city's refuse that will be generated from a particular neighborhood over the course of a year. But the exact time and location of any particular demand for service, unlike scheduled arrivals at a doctor's office, cannot be predicted. Such uncertainties can cause system congestion even when, on the average, there is sufficient system capacity to handle demands.

The need for a probabilistic analysis can be illustrated by a simple example. Suppose that calls for emergency ambulance service are generated from within a community at an average rate of one per hour. For simplicity, we assume that all calls are serviced by the city's *single* ambulance, which is stationed at the city's hospital. Any calls that occur while the ambulance is busy on a previous call are entered into a queue that is depleted in a first-come, first-served manner. In addition, let us assume that the average time to service each call (i.e., the travel time to the scene, on-scene service time, travel time back to the hospital) is 1/2 hour and the standard deviation is $\sqrt{2}/4 \approx 0.354$ hour. In this illustrative example we are ignoring complications due to meal breaks, serious accidents requiring multiple ambulances, and possible time-varying demand rates.

Now, *deterministic reasoning*, which is not uncommon in present planning procedures, argues that this ambulance is *not* overloaded because it is busy on ambulance runs only 4 hours out of each 8-hour tour of duty. After all, the unit is being utilized at only one-half "capacity." The same reasoning argues that since the ambulance services calls twice as fast as they arrive, any queueing delay incurred by arriving calls should be negligibly small. Finally, using deterministic arguments, one should expect the number of calls generated during any 8-hour tour to be very close to the average, eight.

Switching to probabilistic arguments, it has been found that the generation of emergency calls in a city can be accurately modeled by the Poisson process, a random process to be reviewed in Chapter 2. The Poisson process arises when specific postulates governing microscopic system behavior are satisfied. Nearly all of those apply to urban emergency services. Given the Poisson assumptions, the number of ambulance calls that occur during any particular 8-hour tour is a random variable with the Poisson probability mass function,

$$P(n) \equiv \text{probability that exactly } n \text{ ambulance calls occur during an 8-hour tour}$$

$$= \frac{(8)^n e^{-8}}{n!} \qquad n = 0, 1, 2, \ldots$$

This distribution is plotted in Figure 1.2. The probability of *exactly* eight calls arriving during a tour is only 0.1396. For nearly 20 percent of the tours, the number of calls will be less than or equal to *five*, and for another almost 20 percent of the tours, the number of calls will be greater than or equal to *eleven*. Thus, in an average 5-day workweek, the typical random fluctuations in the Poisson process will generate one "light-workload" day ($n \leq 5$) and one "heavy-workload" day ($n \geq 11$). But few weeks are precisely average, as some will have several light-workload days, others several heavy-workload

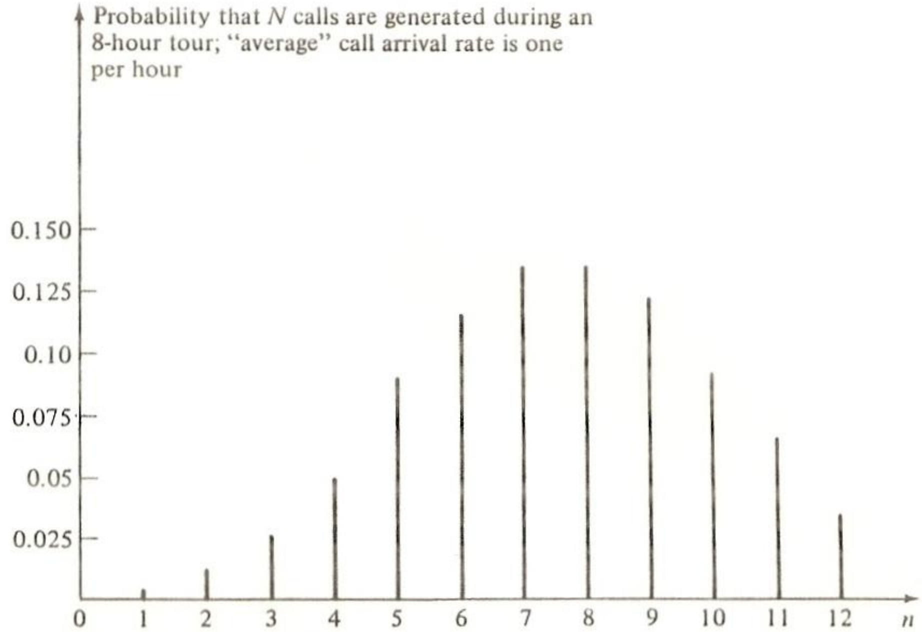

FIGURE 1.2 The Poisson probability mass function.

days. True, the *average* workload per tour, measured in hours required to service calls, is 4 hours. But one can show that there is considerable dispersion about this average, as measured by a standard deviation of $\sqrt{3} \approx 1.73$ hours. Finally, using $M/G/1$ queueing theory (Chapter 4), one finds that fully 50 percent of the calls arrive while the ambulance is busy and thus have to be stacked in the queue; the *average time* spent in the queue by these calls is 45 minutes, reflecting a truly congested system. These insights illustrate the usefulness of a probabilistic analysis.

To implement the results of a probabilistic analysis, it is necessary that operational objectives be stated in compatible terms. For instance, the following statements exhibit probabilistic points of view: "Allocate sufficiently many ambulances so that *85 percent* of emergency calls for ambulances can be answered without queueing delay," or "so that the *average* travel time to ambulance calls is 5 minutes." Although such explicit target levels for performance may be difficult to state (see Chapter 8), operating systems reveal *implicit* decisions that could be quite unacceptable if made clear to an agency administrator.

We are now ready to begin our tour of models and methods for the analysis of urban service systems. Given the central role of uncertainty in the operation of these systems, our first concern deals with the fundamentals of probabilistic modeling.