

**Seção 5.6**

1. (a)  $x = 0$ ; (b)  $r(r - 1) = 0$ ;  $r_1 = 1, r_2 = 0$
2. (a)  $x = 0$ ; (b)  $r^2 - 3r + 2 = 0$ ;  $r_1 = 2, r_2 = 1$
3. (a)  $x = 0$ ; (b)  $r(r - 1) = 0$ ;  $r_1 = 1, r_2 = 0$   
(a)  $x = 1$ ; (b)  $r(r + 5) = 0$ ;  $r_1 = 0, r_2 = -5$
4. Não tem ponto singular regular
5. (a)  $x = 0$ ;  
(b)  $r^2 + 2r - 2 = 0$ ;  $r_1 = -1 + \sqrt{3} \cong 0,732, r_2 = -1 - \sqrt{3} \cong -2,73$
6. (a)  $x = 0$ ; (b)  $r(r - \frac{3}{4}) = 0$ ;  $r_1 = \frac{3}{4}, r_2 = 0$   
(a)  $x = -2$ ; (b)  $r(r - \frac{5}{4}) = 0$ ;  $r_1 = \frac{5}{4}, r_2 = 0$
7. (a)  $x = 0$ ; (b)  $r^2 + 1 = 0$ ;  $r_1 = i, r_2 = -i$
8. (a)  $x = -1$ ;  
(b)  $r^2 - 7r + 3 = 0$ ;  $r_1 = (7 + \sqrt{37})/2 \cong 6,54, r_2 = (7 - \sqrt{37})/2 \cong 0,459$
9. (a)  $x = 1$ ; (b)  $r^2 + r = 0$ ;  $r_1 = 0, r_2 = -1$
10. (a)  $x = -2$ ; (b)  $r^2 - (5/4)r = 0$ ;  $r_1 = 5/4, r_2 = 0$
11. (a)  $x = 2$ ; (b)  $r^2 - 2r = 0$ ;  $r_1 = 2, r_2 = 0$   
(a)  $x = -2$ ; (b)  $r^2 - 2r = 0$ ;  $r_1 = 2, r_2 = 0$
12. (a)  $x = 0$ ; (b)  $r^2 - (5/3)r = 0$ ;  $r_1 = 5/3, r_2 = 0$   
(a)  $x = -3$ ; (b)  $r^2 - (r/3) - 1 = 0$ ;  $r_1 = (1 + \sqrt{37})/6 \cong 1,18, r_2 = (1 - \sqrt{37})/6 \cong -0,847$
13. (b)  $r_1 = 0, r_2 = 0$   
(c)  $y_1(x) = 1 + x + \frac{1}{4}x^2 + \frac{1}{36}x^3 + \dots$   
 $y_2(x) = y_1(x) \ln x - 2x - \frac{3}{4}x^2 - \frac{11}{108}x^3 + \dots$
14. (b)  $r_1 = 1, r_2 = 0$   
(c)  $y_1(x) = x - 4x^2 + \frac{17}{3}x^3 - \frac{47}{12}x^4 + \dots$   
 $y_2(x) = -6y_1(x) \ln x + 1 - 33x^2 + \frac{449}{6}x^3 + \dots$
15. (b)  $r_1 = 1, r_2 = 0$   
(c)  $y_1(x) = x + \frac{3}{2}x^2 + \frac{9}{4}x^3 + \frac{51}{16}x^4 + \dots$   
 $y_2(x) = 3y_1(x) \ln x + 1 - \frac{21}{4}x^2 - \frac{19}{4}x^3 + \dots$
16. (b)  $r_1 = 1, r_2 = 0$   
(c)  $y_1(x) = x - \frac{1}{2}x^2 + \frac{1}{12}x^3 - \frac{1}{144}x^4 + \dots$   
 $y_2(x) = -y_1(x) \ln x + 1 - \frac{3}{4}x^2 + \frac{7}{36}x^3 - \frac{35}{1728}x^4 + \dots$
17. (b)  $r_1 = 1, r_2 = -1$   
(c)  $y_1(x) = x - \frac{1}{24}x^3 + \frac{1}{720}x^5 + \dots$   
 $y_2(x) = -\frac{1}{3}y_1(x) \ln x + x^{-1} - \frac{1}{90}x^3 + \dots$
18. (b)  $r_1 = \frac{1}{2}, r_2 = 0$   
(c)  $y_1(x) = (x - 1)^{1/2}[1 - \frac{3}{4}(x - 1) + \frac{53}{480}(x - 1)^2 + \dots]$ , (d)  $\rho = 1$
19. (c) Sugestão:  $(n - 1)(n - 2) + (1 + \alpha + \beta)(n - 1) + \alpha\beta = (n - 1 + \alpha)(n - 1 + \beta)$   
(d) Sugestão:  $(n - \gamma)(n - 1 - \gamma) + (1 + \alpha + \beta)(n - \gamma) + \alpha\beta = (n - \gamma + \alpha)(n - \gamma + \beta)$

**Seção 5.7**

1.  $y_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!(n+1)!}$   
 $y_2(x) = -y_1(x) \ln x + \frac{1}{x} \left[ 1 - \sum_{n=1}^{\infty} \frac{H_n + H_{n-1}}{n!(n-1)!} (-1)^n x^n \right]$
2.  $y_1(x) = \frac{1}{x} \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(n!)^2}, \quad y_2(x) = y_1(x) \ln x - \frac{2}{x} \sum_{n=1}^{\infty} \frac{(-1)^n H_n x^n}{(n!)^2}$
3.  $y_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{(n!)^2} x^n, \quad y_2(x) = y_1(x) \ln x - 2 \sum_{n=1}^{\infty} \frac{(-1)^n 2^n H_n x^n}{(n!)^2}$
4.  $y_1(x) = \frac{1}{x} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+1)!} x^n$   
 $y_2(x) = -y_1(x) \ln x + \frac{1}{x^2} \left[ 1 - \sum_{n=1}^{\infty} \frac{H_n + H_{n-1}}{n!(n-1)!} (-1)^n x^n \right]$

$$5. y_1(x) = x^{3/2} \left[ 1 + \sum_{m=1}^{\infty} \frac{(-1)^m}{m!(1+\frac{3}{2})(2+\frac{3}{2})\cdots(m+\frac{3}{2})} \left(\frac{x}{2}\right)^{2m} \right]$$

$$y_2(x) = x^{-3/2} \left[ 1 + \sum_{m=1}^{\infty} \frac{(-1)^m}{m!(1-\frac{3}{2})(2-\frac{3}{2})\cdots(m-\frac{3}{2})} \left(\frac{x}{2}\right)^{2m} \right]$$

Sugestão: faça  $n = 2m$  na relação de recorrência,  $m = 1, 2, 3, \dots$

Para  $r = -\frac{3}{2}$ ,  $a_1 = 0$  e  $a_3$  é arbitrário.

## CAPÍTULO 6

### Seção 6.1

1. Seccionalmente contínua
2. Nenhuma das duas
3. Contínua
4. Seccionalmente contínua
5. (a)  $F(s) = 1/s^2$ ,  $s > 0$       (b)  $F(s) = 2/s^3$ ,  $s > 0$   
 (c)  $F(s) = n!/s^{n+1}$ ,  $s > 0$
6.  $F(s) = s/(s^2 + a^2)$ ,  $s > 0$
7.  $F(s) = \frac{s}{s^2 - b^2}$ ,  $s > |b|$
8.  $F(s) = \frac{b}{s^2 - b^2}$ ,  $s > |b|$
9.  $F(s) = \frac{s-a}{(s-a)^2 - b^2}$ ,  $s-a > |b|$
10.  $F(s) = \frac{b}{(s-a)^2 - b^2}$ ,  $s-a > |b|$
11.  $F(s) = \frac{b}{s^2 + b^2}$ ,  $s > 0$
12.  $F(s) = \frac{s}{s^2 + b^2}$ ,  $s > 0$
13.  $F(s) = \frac{b}{(s-a)^2 + b^2}$ ,  $s > a$
14.  $F(s) = \frac{s-a}{(s-a)^2 + b^2}$ ,  $s > a$
15.  $F(s) = \frac{1}{(s-a)^2}$ ,  $s > a$
16.  $F(s) = \frac{2as}{(s^2 + a^2)^2}$ ,  $s > 0$
17.  $F(s) = \frac{s^2 + a^2}{(s-a)^2(s+a)^2}$ ,  $s > |a|$
18.  $F(s) = \frac{n!}{(s-a)^{n+1}}$ ,  $s > a$
19.  $F(s) = \frac{2a(3s^2 + a^2)}{(s^2 + a^2)^3}$ ,  $s > 0$
20.  $F(s) = \frac{2a(3s^2 + a^2)}{(s^2 - a^2)^3}$ ,  $s > |a|$
21. Converge
22. Converge
23. Diverge
24. Converge
26. (d)  $\Gamma(3/2) = \sqrt{\pi}/2$ ;  $\Gamma(11/2) = 945\sqrt{\pi}/32$

### Seção 6.2

1.  $f(t) = \frac{3}{2} \sin 2t$
2.  $f(t) = 2t^2 e^t$
3.  $f(t) = \frac{2}{3} e^t - \frac{2}{3} e^{-4t}$
4.  $f(t) = \frac{9}{8} e^{3t} + \frac{6}{8} e^{-2t}$
5.  $f(t) = 2e^{-t} \cos 2t$
6.  $f(t) = 2 \cosh 2t - \frac{3}{2} \sinh 2t$
7.  $f(t) = 2e^t \cos t + 3e^t \sin t$
8.  $f(t) = 3 - 2 \sin 2t + 5 \cos 2t$
9.  $f(t) = -2e^{-2t} \cos t + 5e^{-2t} \sin t$
10.  $f(t) = 2e^{-t} \cos 3t - \frac{5}{3} e^{-t} \sin 3t$
11.  $y = \frac{1}{3}(e^{3t} + 4e^{-2t})$
12.  $y = 2e^{-t} - e^{-2t}$
13.  $y = e^t \sin t$
14.  $y = e^{2t} - te^{2t}$
15.  $y = 2e^t \cos \sqrt{3}t - (2/\sqrt{3})e^t \sin \sqrt{3}t$
16.  $y = 2e^{-t} \cos 2t + \frac{1}{2}e^{-t} \sin 2t$
17.  $y = te^t - t^2 e^t + \frac{2}{3}t^3 e^t$
18.  $y = \cosh t$
19.  $y = \cos \sqrt{2}t$
20.  $y = (\omega^2 - 4)^{-1}[(\omega^2 - 5) \cos \omega t + \cos 2t]$
21.  $y = \frac{1}{5}(\cos t - 2 \sin t + 4e^t \cos t - 2e^t \sin t)$
22.  $y = \frac{1}{5}(e^{-t} - e^t \cos t + 7e^t \sin t)$
23.  $y = 2e^{-t} + te^{-t} + 2t^2 e^{-t}$
24.  $Y(s) = \frac{s}{s^2 + 4} + \frac{1 - e^{-\pi s}}{s(s^2 + 4)}$
25.  $Y(s) = \frac{1}{s^2(s^2 + 1)} - \frac{e^{-s}(s+1)}{s^2(s^2 + 1)}$
26.  $Y(s) = (1 - e^{-s})/s^2(s^2 + 4)$
29.  $F(s) = 1/(s-a)^2$
30.  $F(s) = 2b(3s^2 - b^2)/(s^2 + b^2)^3$
31.  $F(s) = n!/s^{n+1}$
32.  $F(s) = n!/(s-a)^{n+1}$
33.  $F(s) = 2b(s-a)/[(s-a)^2 + b^2]^2$
34.  $F(s) = [(s-a)^2 - b^2]/[(s-a)^2 + b^2]^2$
36. (a)  $Y' + s^2 Y = s$       (b)  $s^2 Y'' + 2sY' - [s^2 + \alpha(\alpha+1)]Y = -1$

### Seção 6.3

7. (b)  $f(t) = -2u_3(t) + 4u_5(t) - u_7(t)$
8. (b)  $f(t) = 1 - 2u_1(t) + 2u_2(t) - 2u_3(t) + u_4(t)$

9. (b)  $f(t) = 1 + u_2(t)[e^{-(t-2)} - 1]$
11. (b)  $f(t) = t - u_1(t) - u_2(t) - u_3(t)(t-2)$
12. (b)  $f(t) = t + u_2(t)(2-t) + u_5(t)(5-t) - u_7(t)(7-t)$
13.  $F(s) = 2e^{-s}/s^3$
15.  $F(s) = \frac{e^{-\pi s}}{s^2} - \frac{e^{-2\pi s}}{s^2}(1+\pi s)$
17.  $F(s) = s^{-2}[(1-s)e^{-2s} - (1+s)e^{-3s}]$
19.  $f(t) = t^3 e^{2t}$
21.  $f(t) = 2u_2(t)e^{t-2} \cos(t-2)$
23.  $f(t) = u_1(t)e^{2(t-1)} \cosh(t-1)$
26.  $f(t) = 2(2t)^n$
28.  $f(t) = \frac{1}{6}e^{t/3}(e^{2t/3}-1)$
30.  $F(s) = s^{-1}(1-e^{-s}), \quad s > 0$
32.  $F(s) = \frac{1}{s}[1-e^{-s} + \dots + e^{-2ns} - e^{-(2n+1)s}] = \frac{1-e^{-(2n+2)s}}{s(1+e^{-s})}, \quad s > 0$
33.  $F(s) = \frac{1}{s} \sum_{n=0}^{\infty} (-1)^n e^{-ns} = \frac{1/s}{1+e^{-s}}, \quad s > 0$
35.  $\mathcal{L}\{f(t)\} = \frac{1/s}{1+e^{-s}}, \quad s > 0$
37.  $\mathcal{L}\{f(t)\} = \frac{1-(1+s)e^{-s}}{s^2(1-e^{-s})}, \quad s > 0$
39. (a)  $\mathcal{L}\{f(t)\} = s^{-1}(1-e^{-s}), \quad s > 0$   
 (b)  $\mathcal{L}\{g(t)\} = s^{-2}(1-e^{-s}), \quad s > 0$   
 (c)  $\mathcal{L}\{h(t)\} = s^{-2}(1-e^{-s})^2, \quad s > 0$
40. (b)  $\mathcal{L}\{p(t)\} = \frac{1-e^{-s}}{s^2(1+e^{-s})}, \quad s > 0$
10. (b)  $f(t) = t^2 + u_2(t)(1-t^2)$
14.  $F(s) = e^{-s}(s^2+2)/s^3$
16.  $F(s) = \frac{1}{s}(e^{-s}+2e^{-3s}-6e^{-4s})$
18.  $F(s) = (1-e^{-s})/s^2$
20.  $f(t) = \frac{1}{3}u_2(t)[e^{t-2} - e^{-2(t-2)}]$
22.  $f(t) = u_2(t) \sinh 2(t-2)$
24.  $f(t) = u_1(t) + u_2(t) - u_3(t) - u_4(t)$
27.  $f(t) = \frac{1}{2}e^{-t/2} \cos t$
29.  $f(t) = \frac{1}{2}e^{t/2}u_2(t/2)$
31.  $F(s) = s^{-1}(1-e^{-s}+e^{-2s}-e^{-3s}), \quad s > 0$
36.  $\mathcal{L}\{f(t)\} = \frac{1-e^{-s}}{s(1+e^{-s})}, \quad s > 0$
38.  $\mathcal{L}\{f(t)\} = \frac{1+e^{-\pi s}}{(1+s^2)(1-e^{-\pi s})}, \quad s > 0$

#### Seção 6.4

1. (a)  $y = 1 - \cos t + \sin t - u_{3\pi}(t)(1 + \cos t)$
2. (a)  $y = e^{-t} \sin t + \frac{1}{2}u_{\pi}(t)[1 + e^{-(t-\pi)} \cos t + e^{-(t-\pi)} \sin t] - \frac{1}{2}u_{2\pi}(t)[1 - e^{-(t-2\pi)} \cos t - e^{-(t-2\pi)} \sin t]$
3. (a)  $y = \frac{1}{6}[1 - u_{2\pi}(t)](2 \sin t - \sin 2t)$
4. (a)  $y = \frac{1}{6}(2 \sin t - \sin 2t) - \frac{1}{6}u_{\pi}(t)(2 \sin t + \sin 2t)$
5. (a)  $y = \frac{1}{2} + \frac{1}{2}e^{-2t} - e^{-t} - u_{10}(t)[\frac{1}{2} + \frac{1}{2}e^{-2(t-10)} - e^{-(t-10)}]$
6. (a)  $y = e^{-t} - e^{-2t} + u_2(t)[\frac{1}{2} - e^{-(t-2)} + \frac{1}{2}e^{-2(t-2)}]$
7. (a)  $y = \cos t + u_{3\pi}(t)[1 - \cos(t-3\pi)]$
8. (a)  $y = h(t) - u_{\pi/2}(t)h(t-\pi/2), \quad h(t) = \frac{4}{25}(-4 + 5t + 4e^{-t/2} \cos t - 3e^{-t/2} \sin t)$
9. (a)  $y = \frac{1}{2} \sin t + \frac{1}{2}t - \frac{1}{2}u_6(t)[t-6 - \sin(t-6)]$
10. (a)  $y = h(t) + u_{\pi}(t)h(t-\pi), \quad h(t) = \frac{4}{17}[-4 \cos t + \sin t + 4e^{-t/2} \cos t + e^{-t/2} \sin t]$
11. (a)  $y = u_{\pi}(t)[\frac{1}{4} - \frac{1}{4} \cos(2t-2\pi)] - u_{3\pi}(t)[\frac{1}{4} - \frac{1}{4} \cos(2t-6\pi)]$
12. (a)  $y = u_1(t)h(t-1) - u_2(t)h(t-2), \quad h(t) = -1 + (\cos t + \cosh t)/2$
13. (a)  $y = h(t) - u_{\pi}(t)h(t-\pi), \quad h(t) = (3 - 4 \cos t + \cos 2t)/12$
14.  $f(t) = [u_{t_0}(t)(t-t_0) - u_{t_0+k}(t)(t-t_0-k)](h/k)$
15.  $g(t) = [u_{t_0}(t)(t-t_0) - 2u_{t_0+k}(t)(t-t_0-k) + u_{t_0+2k}(t)(t-t_0-2k)](h/k)$
16. (b)  $u(t) = 4ku_{3/2}(t)h(t-\frac{3}{2}) - 4ku_{5/2}(t)h(t-\frac{5}{2}),$   
 $h(t) = \frac{1}{4} - (\sqrt{7}/84)e^{-t/8} \sin(3\sqrt{7}t/8) - \frac{1}{4}e^{-t/8} \cos(3\sqrt{7}t/8)$   
 (d)  $k = 2,51$       (e)  $\tau = 25,6773$
17. (a)  $k = 5$   
 (b)  $y = [u_5(t)h(t-5) - u_{5+k}(t)h(t-5-k)]/k, \quad h(t) = \frac{1}{4}t - \frac{1}{8} \sin 2t$
18. (b)  $f_k(t) = [u_{4-k}(t) - u_{4+k}(t)]/2k;$   
 $y = [u_{4-k}(t)h(t-4+k) - u_{4+k}(t)h(t-4-k)]/2k,$   
 $h(t) = \frac{1}{4} - \frac{1}{4}e^{-t/6} \cos(\sqrt{143}t/6) - (\sqrt{143}/572)e^{-t/6} \sin(\sqrt{143}t/6)$
19. (b)  $y = 1 - \cos t + 2 \sum_{k=1}^n (-1)^k u_{k\pi}(t)[1 - \cos(t-k\pi)]$
21. (b)  $y = 1 - \cos t + \sum_{k=1}^n (-1)^k u_{k\pi}(t)[1 - \cos(t-k\pi)]$
23. (a)  $y = 1 - \cos t + 2 \sum_{k=1}^n (-1)^k u_{11k/4}(t)[1 - \cos(t-11k/4)]$

**Seção 6.5**

1. (a)  $y = e^{-t} \cos t + e^{-t} \sin t + u_{\pi}(t)e^{-(t-\pi)} \sin(t-\pi)$
2. (a)  $y = \frac{1}{2}u_{\pi}(t) \sin 2(t-\pi) - \frac{1}{2}u_{2\pi}(t) \sin 2(t-2\pi)$
3. (a)  $y = -\frac{1}{2}e^{-2t} + \frac{1}{2}e^{-t} + u_5(t)[-e^{-2(t-5)} + e^{-(t-5)}] + u_{10}(t)[\frac{1}{2} + \frac{1}{2}e^{-2(t-10)} - e^{-(t-10)}]$
4. (a)  $y = \cosh(t) - 20u_3(t) \operatorname{senh}(t-3)$
5. (a)  $y = \frac{1}{4} \sin t - \frac{1}{4} \cos t + \frac{1}{4}e^{-t} \cos \sqrt{2}t + (1/\sqrt{2})u_{3\pi}(t)e^{-(t-3\pi)} \sin \sqrt{2}(t-3\pi)$
6. (a)  $y = \frac{1}{2} \cos 2t + \frac{1}{2}u_{4\pi}(t) \sin 2(t-4\pi)$
7. (a)  $y = \sin t + u_{2\pi}(t) \sin(t-2\pi)$
8. (a)  $y = u_{\pi/4}(t) \sin 2(t-\pi/4)$
9. (a)  $y = u_{\pi/2}(t)[1 - \cos(t-\pi/2)] + 3u_{3\pi/2}(t) \sin(t-3\pi/2) - u_{2\pi}(t)[1 - \cos(t-2\pi)]$
10. (a)  $y = (1/\sqrt{31})u_{\pi/6}(t) \exp[-\frac{1}{4}(t-\pi/6)] \sin(\sqrt{31}/4)(t-\pi/6)$
11. (a)  $y = \frac{1}{5} \cos t + \frac{2}{5} \sin t - \frac{1}{5}e^{-t} \cos t - \frac{3}{5}e^{-t} \sin t + u_{\pi/2}(t)e^{-(t-\pi/2)} \sin(t-\pi/2)$
12. (a)  $y = u_1(t)[\operatorname{senh}(t-1) - \sin(t-1)]/2$
13. (a)  $-e^{-T/4}\delta(t-5-T)$ ,  $T = 8\pi/\sqrt{15}$
14. (a)  $y = (4/\sqrt{15})u_1(t)e^{-(t-1)/4} \sin(\sqrt{15}/4)(t-1)$ 
  - (b)  $t_1 \cong 2,3613$ ,  $y_1 \cong 0,71153$
  - (c)  $y = (8\sqrt{7}/21)u_1(t)e^{-(t-1)/8} \sin(3\sqrt{7}/8)(t-1)$ ;  $t_1 \cong 2,4569$ ,  $y_1 \cong 0,83351$
  - (d)  $t_1 = 1 + \pi/2 \cong 2,5708$ ,  $y_1 = 1$
15. (a)  $k_1 \cong 2,8108$       (b)  $k_1 \cong 2,3995$       (c)  $k_1 = 2$
16. (a)  $\phi(t, k) = [u_{4-k}(t)h(t-4+k) - u_{4+k}(t)h(t-4-k)]/2k$ ,  $h(t) = 1 - \cos t$ 
  - (b)  $\phi_0(t) = u_4(t) \sin(t-4)$       (c) Sim
17. (b)  $y = \sum_{k=1}^{20} u_{k\pi}(t) \sin(t-k\pi)$       18. (b)  $y = \sum_{k=1}^{20} (-1)^{k+1} u_{k\pi}(t) \sin(t-k\pi)$
19. (b)  $y = \sum_{k=1}^{20} u_{k\pi/2}(t) \sin(t-k\pi/2)$       20. (b)  $y = \sum_{k=1}^{20} (-1)^{k+1} u_{k\pi/2}(t) \sin(t-k\pi/2)$
21. (b)  $y = \sum_{k=1}^{15} u_{(2k-1)\pi}(t) \sin[t-(2k-1)\pi]$       22. (b)  $y = \sum_{k=1}^{40} (-1)^{k+1} u_{11k/4}(t) \sin(t-11k/4)$
23. (b)  $y = \frac{20}{\sqrt{399}} \sum_{k=1}^{20} (-1)^{k+1} u_{k\pi}(t) e^{-(t-k\pi)/20} \sin[\sqrt{399}(t-k\pi)/20]$
24. (b)  $y = \frac{20}{\sqrt{399}} \sum_{k=1}^{15} u_{(2k-1)\pi}(t) e^{-[t-(2k-1)\pi]/20} \sin[\sqrt{399}[t-(2k-1)\pi]/20]$

**Seção 6.6**

3.  $\sin t * \sin t = \frac{1}{2}(\sin t - t \cos t)$  é negativo quando  $t = 2\pi$ , por exemplo.
4.  $F(s) = 2/s^2(s^2 + 4)$       5.  $F(s) = 1/(s+1)(s^2 + 1)$
6.  $F(s) = 1/s^2(s-1)$       7.  $F(s) = s/(s^2 + 1)^2$
8.  $f(t) = \frac{1}{6} \int_0^t (t-\tau)^3 \sin \tau d\tau$       9.  $f(t) = \int_0^t e^{-(t-\tau)} \cos 2\tau d\tau$
10.  $f(t) = \frac{1}{2} \int_0^t (t-\tau)e^{-(t-\tau)} \sin 2\tau d\tau$       11.  $f(t) = \int_0^t \sin(t-\tau)g(\tau) d\tau$
12. (c)  $\int_0^1 u'''(1-u)^n du = \frac{\Gamma(m+1)\Gamma(n+1)}{\Gamma(m+n+2)}$
13.  $y = \frac{1}{\omega} \sin \omega t + \frac{1}{\omega} \int_0^t \sin \omega(t-\tau)g(\tau) d\tau$       14.  $y = \int_0^t e^{-(t-\tau)} \sin(t-\tau) \sin \alpha \tau d\tau$
15.  $y = \frac{1}{8} \int_0^t e^{-(t-\tau)/2} \sin 2(t-\tau)g(\tau) d\tau$
16.  $y = e^{-t/2} \cos t - \frac{1}{2}e^{-t/2} \sin t + \int_0^t e^{-(t-\tau)/2} \sin(t-\tau)[1-u_{\pi}(\tau)] d\tau$
17.  $y = 2e^{-2t} + te^{-2t} + \int_0^t (t-\tau)e^{-2(t-\tau)}g(\tau) d\tau$
18.  $y = 2e^{-t} - e^{-2t} + \int_0^t [e^{-(t-\tau)} - e^{-2(t-\tau)}] \cos \alpha \tau d\tau$
19.  $y = \frac{1}{2} \int_0^t [\operatorname{senh}(t-\tau) - \sin(t-\tau)]g(\tau) d\tau$

20.  $y = \frac{4}{3} \cos t - \frac{1}{3} \cos 2t + \frac{1}{6} \int_0^t [2 \sin(t-\tau) - \sin 2(t-\tau)]g(\tau) d\tau$
21.  $\Phi(s) = \frac{F(s)}{1+K(s)}$
22. (a)  $\phi(t) = \frac{1}{3}(4 \sin 2t - 2 \sin t)$
23. (a)  $\phi(t) = \cos t$   
(b)  $\phi''(t) + \phi(t) = 0, \quad \phi(0) = 1, \quad \phi'(0) = 0$
24. (a)  $\phi(t) = \cosh(t)$   
(b)  $\phi''(t) - \phi(t) = 0, \quad \phi(0) = 1, \quad \phi'(0) = 0$
25. (a)  $\phi(t) = (1 - 2t + t^2)e^{-t}$   
(b)  $\phi''(t) + 2\phi'(t) + \phi(t) = 2e^{-t}, \quad \phi(0) = 1, \quad \phi'(0) = -3$
26. (a)  $\phi(t) = \frac{1}{3}e^{-t} - \frac{1}{3}e^{t/2} \cos(\sqrt{3}t/2) + \frac{1}{\sqrt{3}}e^{t/2} \sin(\sqrt{3}t/2)$   
(b)  $\phi'''(t) + \phi(t) = 0, \quad \phi(0) = 0, \quad \phi'(0) = 0, \quad \phi''(0) = 1$
27. (a)  $\phi(t) = \cos t$   
(b)  $\phi^{(4)}(t) - \phi(t) = 0, \quad \phi(0) = 1, \quad \phi'(0) = 0, \quad \phi''(0) = -1, \quad \phi'''(0) = 0$
28. (a)  $\phi(t) = 1 - \frac{2}{\sqrt{3}}e^{-t/2} \sin(\sqrt{3}t/2)$   
(b)  $\phi'''(t) + \phi''(t) + \phi'(t) = 0, \quad \phi(0) = 1, \quad \phi'(0) = -1, \quad \phi''(0) = 1$

**CAPÍTULO 7** Seção 7.1

1.  $x'_1 = x_2, \quad x'_2 = -2x_1 - 0,5x_2$       2.  $x'_1 = x_2, \quad x'_2 = -2x_1 - 0,5x_2 + 3 \sin t$
3.  $x'_1 = x_2, \quad x'_2 = -(1 - 0,25t^{-2})x_1 - t^{-1}x_2$       4.  $x'_1 = x_2, \quad x'_2 = x_3, \quad x'_3 = x_4, \quad x'_4 = x_1$
5.  $x'_1 = x_2, \quad x'_2 = -4x_1 - 0,25x_2 + 2 \cos 3t, \quad x_1(0) = 1, \quad x_2(0) = -2$
6.  $x'_1 = x_2, \quad x'_2 = -q(t)x_1 - p(t)x_2 + g(t); \quad x_1(0) = u_0, \quad x_2(0) = u'_0$
7. (a)  $x_1 = c_1 e^{-t} + c_2 e^{-3t}, \quad x_2 = c_1 e^{-t} - c_2 e^{-3t}$   
(b)  $c_1 = 5/2, \quad c_2 = -1/2$  na solução em (a)  
(c) O gráfico se aproxima da origem no primeiro quadrante tangente à reta  $x_1 = x_2$ .
8. (a)  $x''_1 - x'_1 - 2x_1 = 0$   
(b)  $x_1 = \frac{11}{3}e^{2t} - \frac{2}{3}e^{-t}, \quad x_2 = \frac{11}{6}e^{2t} - \frac{4}{3}e^{-t}$   
(c) O gráfico é assintótico à reta  $x_1 = 2x_2$  no primeiro quadrante.
9. (a)  $2x''_1 - 5x'_1 + 2x_1 = 0$   
(b)  $x_1 = -\frac{3}{2}e^{t/2} - \frac{1}{2}e^{2t}, \quad x_2 = \frac{3}{2}e^{t/2} - \frac{1}{2}e^{2t}$   
(c) O gráfico é assintótico à reta  $x_1 = x_2$  no terceiro quadrante.
10. (a)  $x''_1 + 3x'_1 + 2x_1 = 0$   
(b)  $x_1 = -7e^{-t} + 6e^{-2t}, \quad x_2 = -7e^{-t} + 9e^{-2t}$   
(c) O gráfico se aproxima da origem no terceiro quadrante tangente à reta  $x_1 = x_2$ .
11. (a)  $x''_1 + 4x_1 = 0$   
(b)  $x_1 = 3 \cos 2t + 4 \sin 2t, \quad x_2 = -3 \sin 2t + 4 \cos 2t$   
(c) O gráfico é um círculo centrado na origem com raio 5 percorrido no sentido horário.
12. (a)  $x''_1 + x'_1 + 4,25x_1 = 0$   
(b)  $x_1 = -2e^{-t/2} \cos 2t + 2e^{-t/2} \sin 2t, \quad x_2 = 2e^{-t/2} \cos 2t + 2e^{-t/2} \sin 2t$   
(c) O gráfico é uma espiral se aproximando da origem no sentido horário.
13.  $LRCI'' + LI' + RI = 0$
14.  $y'_1 = y_3, \quad y'_2 = y_4, \quad m_1y'_3 = -(k_1 + k_2)y_1 + k_2y_2 + F_1(t),$   
 $m_2y'_4 = k_2y_1 - (k_2 + k_3)y_2 + F_2(t)$
22. (a)  $Q'_1 = \frac{3}{2} - \frac{1}{10}Q_1 + \frac{3}{40}Q_2, \quad Q_1(0) = 25$   
 $Q'_2 = 3 + \frac{1}{10}Q_1 - \frac{1}{5}Q_2, \quad Q_2(0) = 15$   
(b)  $Q_1^E = 42, \quad Q_2^E = 36$   
(c)  $x'_1 = -\frac{1}{10}x_1 + \frac{3}{40}x_2, \quad x_1(0) = -17$   
 $x'_2 = \frac{1}{10}x_1 - \frac{1}{5}x_2, \quad x_2(0) = -21$
23. (a)  $Q'_1 = 3q_1 - \frac{1}{13}Q_1 + \frac{1}{100}Q_2, \quad Q_1(0) = Q_1^0$   
 $Q'_2 = q_2 + \frac{1}{30}Q_1 - \frac{3}{100}Q_2, \quad Q_2(0) = Q_2^0$   
(b)  $Q_1^E = 6(9q_1 + q_2), \quad Q_2^E = 20(3q_1 + 2q_2)$   
(c) Não  
(d)  $\frac{10}{9} \leq Q_2^E/Q_1^E \leq \frac{20}{3}$

## Seção 7.2

1. (a)  $\begin{pmatrix} 6 & -6 & 3 \\ 5 & 9 & -2 \\ 2 & 3 & 8 \end{pmatrix}$  (b)  $\begin{pmatrix} -15 & 6 & -12 \\ 7 & -18 & -1 \\ -26 & -3 & -5 \end{pmatrix}$

(c)  $\begin{pmatrix} 6 & -12 & 3 \\ 4 & 3 & 7 \\ 9 & 12 & 0 \end{pmatrix}$  (d)  $\begin{pmatrix} -8 & -9 & 11 \\ 14 & 12 & -5 \\ 5 & -8 & 5 \end{pmatrix}$

2. (a)  $\begin{pmatrix} 1-i & -7+2i \\ -1+2i & 2+3i \end{pmatrix}$  (b)  $\begin{pmatrix} 3+4i & 6i \\ 11+6i & 6-5i \end{pmatrix}$

(c)  $\begin{pmatrix} -3+5i & 7+5i \\ 2+i & 7+2i \end{pmatrix}$  (d)  $\begin{pmatrix} 8+7i & 4-4i \\ 6-4i & -4 \end{pmatrix}$

3. (a)  $\begin{pmatrix} -2 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -3 & 1 \end{pmatrix}$  (b)  $\begin{pmatrix} 1 & 3 & -2 \\ 2 & -1 & 1 \\ 3 & -1 & 0 \end{pmatrix}$

(c), (d)  $\begin{pmatrix} -1 & 4 & 0 \\ 3 & -1 & 0 \\ 5 & -4 & 1 \end{pmatrix}$

4. (a)  $\begin{pmatrix} 3-2i & 2-i \\ 1+i & -2+3i \end{pmatrix}$  (b)  $\begin{pmatrix} 3+2i & 1-i \\ 2+i & -2-3i \end{pmatrix}$  (c)  $\begin{pmatrix} 3+2i & 2+i \\ 1-i & -2-3i \end{pmatrix}$

5.  $\begin{pmatrix} 10 & 6 & -4 \\ 0 & 4 & 10 \\ 4 & 4 & 6 \end{pmatrix}$

6. (a)  $\begin{pmatrix} 7 & -11 & -3 \\ 11 & 20 & 17 \\ -4 & 3 & -12 \end{pmatrix}$  (b)  $\begin{pmatrix} 5 & 0 & -1 \\ 2 & 7 & 4 \\ -1 & 1 & 4 \end{pmatrix}$

(c)  $\begin{pmatrix} 6 & -8 & -11 \\ 9 & 15 & 6 \\ -5 & -1 & 5 \end{pmatrix}$

8. (a)  $4i$  (b)  $12-8i$  (c)  $2+2i$  (d)  $16$

10.  $\begin{pmatrix} \frac{3}{11} & -\frac{4}{11} \\ \frac{2}{11} & \frac{1}{11} \end{pmatrix}$

11.  $\begin{pmatrix} \frac{1}{6} & \frac{1}{12} \\ -\frac{1}{2} & \frac{1}{4} \end{pmatrix}$

12.  $\begin{pmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{pmatrix}$

13.  $\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$

14. Singular

15.  $\begin{pmatrix} \frac{1}{2} & -\frac{1}{4} & \frac{1}{8} \\ 0 & \frac{1}{2} & -\frac{1}{4} \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$

16.  $\begin{pmatrix} \frac{1}{10} & \frac{3}{10} & \frac{1}{10} \\ -\frac{2}{10} & \frac{4}{10} & -\frac{2}{10} \\ -\frac{7}{10} & -\frac{1}{10} & \frac{3}{10} \end{pmatrix}$

17. Singular

18.  $\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$

19.  $\begin{pmatrix} 6 & \frac{13}{5} & -\frac{8}{5} & \frac{2}{5} \\ 5 & \frac{11}{5} & -\frac{6}{5} & \frac{4}{5} \\ 0 & -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ -2 & -\frac{4}{5} & \frac{4}{5} & -\frac{1}{5} \end{pmatrix}$

21. (a)  $\begin{pmatrix} 7e^t & 5e^{-t} & 10e^{2t} \\ -e^t & 7e^{-t} & 2e^{2t} \\ 8e^t & 0 & -e^{2t} \end{pmatrix}$  (b)  $\begin{pmatrix} 2e^{2t}-2+3e^{3t} & 1+4e^{-2t}-e^t & 3e^{3t}+2e^t-e^{4t} \\ 4e^{2t}-1-3e^{3t} & 2+2e^{-2t}+e^t & 6e^{3t}+e^t+e^{4t} \\ -2e^{2t}-3+6e^{3t} & -1+6e^{-2t}-2e^t & -3e^{3t}+3e^t-2e^{4t} \end{pmatrix}$

(c)  $\begin{pmatrix} e^t & -2e^{-t} & 2e^{2t} \\ 2e^t & -e^{-t} & -2e^{2t} \\ -e^t & -3e^{-t} & 4e^{2t} \end{pmatrix}$  (d)  $(e-1) \begin{pmatrix} 1 & 2e^{-1} & \frac{1}{2}(e+1) \\ 2 & e^{-1} & -\frac{1}{2}(e+1) \\ -1 & 3e^{-1} & e+1 \end{pmatrix}$

**Seção 7.3**

1.  $x_1 = -\frac{1}{3}, \quad x_2 = \frac{7}{3}, \quad x_3 = -\frac{1}{3}$

2. Não tem solução

3.  $x_1 = -c, \quad x_2 = c + 1, \quad x_3 = c$ , onde  $c$  é arbitrário

4.  $x_1 = c, \quad x_2 = -c, \quad x_3 = -c$ , onde  $c$  é arbitrário

5.  $x_1 = 0, \quad x_2 = 0, \quad x_3 = 0$

6.  $x_1 = c_1, \quad x_2 = c_2, \quad x_3 = c_1 + 2c_2 + 2$

7. Linearmente independente

8.  $\mathbf{x}^{(1)} - 5\mathbf{x}^{(2)} + 2\mathbf{x}^{(3)} = \mathbf{0}$

9.  $2\mathbf{x}^{(1)} - 3\mathbf{x}^{(2)} + 4\mathbf{x}^{(3)} - \mathbf{x}^{(4)} = \mathbf{0}$

10. Linearmente independente

11.  $\mathbf{x}^{(1)} + \mathbf{x}^{(2)} - \mathbf{x}^{(4)} = \mathbf{0}$

13.  $3\mathbf{x}^{(1)}(t) - 6\mathbf{x}^{(2)}(t) + \mathbf{x}^{(3)}(t) = \mathbf{0}$

14. Linearmente independente

16.  $\lambda_1 = 2, \quad \mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}; \quad \lambda_2 = 4, \quad \mathbf{x}^{(2)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

17.  $\lambda_1 = 1 + 2i, \quad \mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ 1-i \end{pmatrix}; \quad \lambda_2 = 1 - 2i, \quad \mathbf{x}^{(2)} = \begin{pmatrix} 1 \\ 1+i \end{pmatrix}$

18.  $\lambda_1 = -3, \quad \mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}; \quad \lambda_2 = -1, \quad \mathbf{x}^{(2)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

19.  $\lambda_1 = 0, \quad \mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ i \end{pmatrix}; \quad \lambda_2 = 2, \quad \mathbf{x}^{(2)} = \begin{pmatrix} 1 \\ -i \end{pmatrix}$

20.  $\lambda_1 = 2, \quad \mathbf{x}^{(1)} = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}; \quad \lambda_2 = -2, \quad \mathbf{x}^{(2)} = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$

21.  $\lambda_1 = -1/2, \quad \mathbf{x}^{(1)} = \begin{pmatrix} 3 \\ 10 \end{pmatrix}; \quad \lambda_2 = -3/2, \quad \mathbf{x}^{(2)} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

22.  $\lambda_1 = 1, \quad \mathbf{x}^{(1)} = \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}; \quad \lambda_2 = 1 + 2i, \quad \mathbf{x}^{(2)} = \begin{pmatrix} 0 \\ 1 \\ -i \end{pmatrix}; \quad \lambda_3 = 1 - 2i, \quad \mathbf{x}^{(3)} = \begin{pmatrix} 0 \\ 1 \\ i \end{pmatrix}$

23.  $\lambda_1 = 1, \quad \mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}; \quad \lambda_2 = 2, \quad \mathbf{x}^{(2)} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}; \quad \lambda_3 = 3, \quad \mathbf{x}^{(3)} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

24.  $\lambda_1 = 1, \quad \mathbf{x}^{(1)} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}; \quad \lambda_2 = 2, \quad \mathbf{x}^{(2)} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}; \quad \lambda_3 = -1, \quad \mathbf{x}^{(3)} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

25.  $\lambda_1 = -1, \quad \mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix}; \quad \lambda_2 = -1, \quad \mathbf{x}^{(2)} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}; \quad \lambda_3 = 8, \quad \mathbf{x}^{(3)} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

**Seção 7.4**

2. (c)  $W(t) = c \exp \int [p_{11}(t) + p_{22}(t)] dt$

6. (a)  $W(t) = t^2$

(b)  $\mathbf{x}^{(1)}$  e  $\mathbf{x}^{(2)}$  são linearmente independentes em todos os pontos, exceto em  $t = 0$ ; eles são linearmente independentes em todos os intervalos.(c) Pelo menos um coeficiente tem que ser descontínuo em  $t = 0$ .

(d)  $\mathbf{x}' = \begin{pmatrix} 0 & 1 \\ -2t^{-2} & 2t^{-1} \end{pmatrix} \mathbf{x}$

7. (a)  $W(t) = t(t-2)e^t$

(b)  $\mathbf{x}^{(1)}$  e  $\mathbf{x}^{(2)}$  são linearmente independentes em todos os pontos, exceto em  $t = 0$  e  $t = 2$ ; eles são linearmente independentes em todos os intervalos.(c) Pelo menos um coeficiente tem que ser descontínuo em  $t = 0$  e em  $t = 2$ .

(d)  $\mathbf{x}' = \begin{pmatrix} 0 & 1 \\ \frac{2-2t}{t^2-2t} & \frac{t^2-2}{t^2-2t} \end{pmatrix} \mathbf{x}$

**Seção 7.5**

1. (a)  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{2t}$

2. (a)  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 2 \\ 3 \end{pmatrix} e^{-2t}$

3. (a)  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-t}$

4. (a)  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$

5. (a)  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t}$

6. (a)  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{t/2} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$

7. (a)  $\mathbf{x} = c_1 \begin{pmatrix} 3 \\ 4 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

8. (a)  $\mathbf{x} = c_1 \begin{pmatrix} -2 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} -3 \\ 1 \end{pmatrix} e^t$

9.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ i \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ -i \end{pmatrix} e^{2t}$

10.  $\mathbf{x} = c_1 \begin{pmatrix} 2+i \\ -1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-it}$

11.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^t + c_3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t}$

12.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} e^{-t} + c_3 \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} e^{8t}$

13.  $\mathbf{x} = c_1 \begin{pmatrix} 4 \\ -5 \\ -7 \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 3 \\ -4 \\ -2 \end{pmatrix} e^{-t} + c_3 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t}$

14.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} e^{-2t} + c_3 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} e^{3t}$

15.  $\mathbf{x} = -\frac{3}{2} \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{2t} + \frac{7}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}$

16.  $\mathbf{x} = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + \frac{1}{2} \begin{pmatrix} 1 \\ 5 \end{pmatrix} e^{3t}$

17.  $\mathbf{x} = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} e^t + 2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} e^{2t}$

18.  $\mathbf{x} = 6 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} e^t + 3 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} e^{-t} - \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix} e^{4t}$

20.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} t + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} t^{-1}$

21.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} t^2 + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} t^4$

22.  $\mathbf{x} = c_1 \begin{pmatrix} 3 \\ 4 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} t^{-2}$

23.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} t^{-1} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} t^2$

29. (a)  $x'_1 = x_2, \quad x'_2 = -(c/a)x_1 - (b/a)x_2$

30. (a)  $\mathbf{x} = -\frac{55}{8} \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{-t/20} + \frac{29}{8} \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-t/4}$

(c)  $T \cong 74.39$

31. (a)  $\mathbf{x} = c_1 \begin{pmatrix} -\sqrt{2} \\ 1 \end{pmatrix} e^{(-2+\sqrt{2})t/2} + c_2 \begin{pmatrix} \sqrt{2} \\ 1 \end{pmatrix} e^{(-2-\sqrt{2})t/2};$

$r_{1,2} = (-2 \pm \sqrt{2})/2;$  nô

(b)  $\mathbf{x} = c_1 \begin{pmatrix} -1 \\ \sqrt{2} \end{pmatrix} e^{(-1+\sqrt{2})t} + c_2 \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} e^{(-1-\sqrt{2})t}; \quad r_{1,2} = -1 \pm \sqrt{2};$  ponto de sela

(c)  $r_{1,2} = -1 \pm \sqrt{\alpha}; \quad \alpha = 1$

32. (a)  $\begin{pmatrix} I \\ V \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t}$

33. (a)  $\left( \frac{1}{CR_2} - \frac{R_1}{L} \right)^2 - \frac{4}{CL} > 0$

**Seção 7.6**

1. (a)  $\mathbf{x} = c_1 e^t \begin{pmatrix} \cos 2t \\ \cos 2t + \sin 2t \end{pmatrix} + c_2 e^t \begin{pmatrix} \sin 2t \\ -\cos 2t + \sin 2t \end{pmatrix}$

2. (a)  $\mathbf{x} = c_1 e^{-t} \begin{pmatrix} 2 \cos 2t \\ \sin 2t \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} -2 \sin 2t \\ \cos 2t \end{pmatrix}$

3. (a)  $\mathbf{x} = c_1 \begin{pmatrix} 5 \cos t \\ 2 \cos t + \sin t \end{pmatrix} + c_2 \begin{pmatrix} 5 \sin t \\ -\cos t + 2 \sin t \end{pmatrix}$

4. (a)  $\mathbf{x} = c_1 e^{t/2} \begin{pmatrix} 5 \cos \frac{3}{2}t \\ 3(\cos \frac{3}{2}t + \sin \frac{3}{2}t) \end{pmatrix} + c_2 e^{t/2} \begin{pmatrix} 5 \sin \frac{3}{2}t \\ 3(-\cos \frac{3}{2}t + \sin \frac{3}{2}t) \end{pmatrix}$

5. (a)  $\mathbf{x} = c_1 e^{-t} \begin{pmatrix} \cos t \\ 2 \cos t + \sin t \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} \sin t \\ -\cos t + 2 \sin t \end{pmatrix}$

6. (a)  $\mathbf{x} = c_1 \begin{pmatrix} -2 \cos 3t \\ \cos 3t + 3 \sin 3t \end{pmatrix} + c_2 \begin{pmatrix} -2 \sin 3t \\ \sin 3t - 3 \cos 3t \end{pmatrix}$

7.  $\mathbf{x} = c_1 \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix} e^t + c_2 e^t \begin{pmatrix} 0 \\ \cos 2t \\ \sin 2t \end{pmatrix} + c_3 e^t \begin{pmatrix} 0 \\ \sin 2t \\ -\cos 2t \end{pmatrix}$

8.  $\mathbf{x} = c_1 \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} e^{-2t} + c_2 e^{-t} \begin{pmatrix} -\sqrt{2} \sin \sqrt{2}t \\ \cos \sqrt{2}t \\ -\cos \sqrt{2}t - \sqrt{2} \sin \sqrt{2}t \end{pmatrix} + c_3 e^{-t} \begin{pmatrix} \sqrt{2} \cos \sqrt{2}t \\ \sin \sqrt{2}t \\ \sqrt{2} \cos \sqrt{2}t - \sin \sqrt{2}t \end{pmatrix}$

9.  $\mathbf{x} = e^{-t} \begin{pmatrix} \cos t - 3 \sin t \\ \cos t - \sin t \end{pmatrix}$

10.  $\mathbf{x} = e^{-2t} \begin{pmatrix} \cos t - 5 \sin t \\ -2 \cos t - 3 \sin t \end{pmatrix}$

11. (a)  $r = -\frac{1}{4} \pm i$

12. (a)  $r = \frac{1}{5} \pm i$

13. (a)  $r = \alpha \pm i$  (b)  $\alpha = 0$

14. (a)  $r = (\alpha \pm \sqrt{\alpha^2 - 20})/2$  (b)  $\alpha = -\sqrt{20}, 0, \sqrt{20}$

15. (a)  $r = \pm \sqrt{4 - 5\alpha}$  (b)  $\alpha = 4/5$

16. (a)  $r = \frac{5}{4} \pm \frac{1}{2}\sqrt{3\alpha}$  (b)  $\alpha = 0, 25/12$

17. (a)  $r = -1 \pm \sqrt{-\alpha}$  (b)  $\alpha = -1, 0$

18. (a)  $r = -\frac{1}{2} \pm \frac{1}{2}\sqrt{49 - 24\alpha}$  (b)  $\alpha = 2, 49/24$

19. (a)  $r = \frac{1}{2}\alpha - 2 \pm \sqrt{\alpha^2 + 8\alpha - 24}$  (b)  $\alpha = -4 - 2\sqrt{10}, -4 + 2\sqrt{10}, 5/2$

20. (a)  $r = -1 \pm \sqrt{25 + 8\alpha}$  (b)  $\alpha = -25/8, -3$

21.  $\mathbf{x} = c_1 t^{-1} \begin{pmatrix} \cos(\sqrt{2} \ln t) \\ \sqrt{2} \sin(\sqrt{2} \ln t) \end{pmatrix} + c_2 t^{-1} \begin{pmatrix} \sin(\sqrt{2} \ln t) \\ -\sqrt{2} \cos(\sqrt{2} \ln t) \end{pmatrix}$

22.  $\mathbf{x} = c_1 \begin{pmatrix} 5 \cos(\ln t) \\ 2 \cos(\ln t) + \sin(\ln t) \end{pmatrix} + c_2 \begin{pmatrix} 5 \sin(\ln t) \\ -\cos(\ln t) + 2 \sin(\ln t) \end{pmatrix}$

23. (a)  $r = -\frac{1}{4} \pm i, -\frac{1}{4}$

24. (a)  $r = -\frac{1}{4} \pm i, \frac{1}{10}$

25. (b)  $\begin{pmatrix} I \\ V \end{pmatrix} = c_1 e^{-t/2} \begin{pmatrix} \cos(t/2) \\ 4 \sin(t/2) \end{pmatrix} + c_2 e^{-t/2} \begin{pmatrix} \sin(t/2) \\ -4 \cos(t/2) \end{pmatrix}$

(c) Use  $c_1 = 2, c_2 = -\frac{3}{4}$  na resposta do item (b).

(d)  $\lim_{t \rightarrow \infty} I(t) = \lim_{t \rightarrow \infty} V(t) = 0$ ; não

26. (b)  $\begin{pmatrix} I \\ V \end{pmatrix} = c_1 e^{-t} \begin{pmatrix} \cos t \\ -\cos t - \sin t \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} \sin t \\ -\sin t + \cos t \end{pmatrix}$

(c) Use  $c_1 = 2$  e  $c_2 = 3$  na resposta do item (b).

(d)  $\lim_{t \rightarrow \infty} I(t) = \lim_{t \rightarrow \infty} V(t) = 0$ ; não

28. (b)  $r = \pm i\sqrt{k/m}$  (d)  $|r|$  é a frequência natural.

29. (c)  $r_1^2 = -1, \xi^{(1)} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}; r_2^2 = -4, \xi^{(2)} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

(d)  $x_1 = 3c_1 \cos t + 3c_2 \sin t + 3c_3 \cos 2t + 3c_4 \sin 2t,$

$x_2 = 2c_1 \cos t + 2c_2 \sin t - 4c_3 \cos 2t - 4c_4 \sin 2t$

(e)  $x'_1 = -3c_1 \sin t + 3c_2 \cos t - 6c_3 \sin 2t + 6c_4 \cos 2t,$

$x'_2 = -2c_1 \sin t + 2c_2 \cos t + 8c_3 \sin 2t - 8c_4 \cos 2t$

30. (a)  $\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & 3 & 0 & 0 \\ 9/4 & -13/4 & 0 & 0 \end{pmatrix}$

(b)  $r_1 = i, \xi^{(1)} = \begin{pmatrix} 1 \\ 1 \\ i \\ i \end{pmatrix}; r_2 = -i, \xi^{(2)} = \begin{pmatrix} 1 \\ 1 \\ -i \\ -i \end{pmatrix};$

$r_3 = \frac{5}{2}i, \xi^{(3)} = \begin{pmatrix} 4 \\ -3 \\ 10i \\ -\frac{15}{2}i \end{pmatrix}; r_4 = -\frac{5}{2}i, \xi^{(4)} = \begin{pmatrix} 4 \\ -3 \\ -10i \\ \frac{15}{2}i \end{pmatrix}$

(c)  $\mathbf{y} = c_1 \begin{pmatrix} \cos t \\ \cos t \\ -\sin t \\ -\sin t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ \sin t \\ \cos t \\ \cos t \end{pmatrix} + c_3 \begin{pmatrix} 4 \cos \frac{5}{2}t \\ -3 \cos \frac{5}{2}t \\ -10 \sin \frac{5}{2}t \\ \frac{15}{2} \sin \frac{5}{2}t \end{pmatrix} + c_4 \begin{pmatrix} 4 \sin \frac{5}{2}t \\ -3 \sin \frac{5}{2}t \\ 10 \cos \frac{5}{2}t \\ -\frac{15}{2} \cos \frac{5}{2}t \end{pmatrix}$

(e)  $c_1 = \frac{10}{7}, c_2 = 0, c_3 = \frac{1}{7}, c_4 = 0.$  período =  $4\pi.$

31. (a)  $\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 \end{pmatrix}$

(b)  $r_1 = i, \xi^{(1)} = \begin{pmatrix} 1 \\ 1 \\ i \\ i \end{pmatrix}; r_2 = -i, \xi^{(2)} = \begin{pmatrix} 1 \\ 1 \\ -i \\ -i \end{pmatrix};$

$r_3 = \sqrt{3}i, \xi^{(3)} = \begin{pmatrix} 1 \\ -1 \\ \sqrt{3}i \\ -\sqrt{3}i \end{pmatrix}; r_4 = -\sqrt{3}i, \xi^{(4)} = \begin{pmatrix} 1 \\ -1 \\ -\sqrt{3}i \\ \sqrt{3}i \end{pmatrix}$

(c)  $\mathbf{y} = c_1 \begin{pmatrix} \cos t \\ \cos t \\ -\sin t \\ -\sin t \end{pmatrix} + c_2 \begin{pmatrix} \sin t \\ \sin t \\ \cos t \\ \cos t \end{pmatrix} + c_3 \begin{pmatrix} \cos \sqrt{3}t \\ -\cos \sqrt{3}t \\ -\sqrt{3} \sin \sqrt{3}t \\ \sqrt{3} \sin \sqrt{3}t \end{pmatrix} + c_4 \begin{pmatrix} \sin \sqrt{3}t \\ -\sin \sqrt{3}t \\ \sqrt{3} \cos \sqrt{3}t \\ -\sqrt{3} \cos \sqrt{3}t \end{pmatrix}$

(e)  $c_1 = 1, c_2 = 0, c_3 = -2, c_4 = 0.$

### Seção 7.7

1. (b)  $\Phi(t) = \begin{pmatrix} -\frac{1}{3}e^{-t} + \frac{4}{3}e^{2t} & \frac{2}{3}e^{-t} - \frac{2}{3}e^{2t} \\ -\frac{2}{3}e^{-t} + \frac{2}{3}e^{2t} & \frac{4}{3}e^{-t} - \frac{1}{3}e^{2t} \end{pmatrix}$

2. (b)  $\Phi(t) = \begin{pmatrix} \frac{1}{2}e^{-t/2} + \frac{1}{2}e^{-t} & e^{-t/2} - e^{-t} \\ \frac{1}{4}e^{-t/2} - \frac{1}{4}e^{-t} & \frac{1}{2}e^{-t/2} + \frac{1}{2}e^{-t} \end{pmatrix}$

3. (b)  $\Phi(t) = \begin{pmatrix} \frac{3}{2}e^t - \frac{1}{2}e^{-t} & -\frac{1}{2}e^t + \frac{1}{2}e^{-t} \\ \frac{3}{2}e^t - \frac{3}{2}e^{-t} & -\frac{1}{2}e^t + \frac{3}{2}e^{-t} \end{pmatrix}$

4. (b)  $\Phi(t) = \begin{pmatrix} \frac{1}{5}e^{-3t} + \frac{4}{5}e^{2t} & -\frac{1}{5}e^{-3t} + \frac{1}{5}e^{2t} \\ -\frac{4}{5}e^{-3t} + \frac{4}{5}e^{2t} & \frac{4}{5}e^{-3t} + \frac{1}{5}e^{2t} \end{pmatrix}$

5. (b)  $\Phi(t) = \begin{pmatrix} \cos t + 2 \sin t & -5 \sin t \\ \sin t & \cos t - 2 \sin t \end{pmatrix}$

6. (b)  $\Phi(t) = \begin{pmatrix} e^{-t} \cos 2t & -2e^{-t} \sin 2t \\ \frac{1}{2}e^{-t} \sin 2t & e^{-t} \cos 2t \end{pmatrix}$

7. (b)  $\Phi(t) = \begin{pmatrix} -\frac{1}{2}e^{2t} + \frac{3}{2}e^{4t} & \frac{1}{2}e^{2t} - \frac{1}{2}e^{4t} \\ -\frac{3}{2}e^{2t} + \frac{3}{2}e^{4t} & \frac{3}{2}e^{2t} - \frac{1}{2}e^{4t} \end{pmatrix}$

8. (b)  $\Phi(t) = \begin{pmatrix} e^{-t} \cos t + 2e^{-t} \sin t & e^{-t} \sin t \\ 5e^{-t} \sin t & e^{-t} \cos t - 2e^{-t} \sin t \end{pmatrix}$
9. (b)  $\Phi(t) = \begin{pmatrix} -2e^{-2t} + 3e^{-t} & -e^{-2t} + e^{-t} & -e^{-2t} + e^{-t} \\ \frac{5}{2}e^{-2t} - 4e^{-t} + \frac{3}{2}e^{2t} & \frac{5}{4}e^{-2t} - \frac{4}{3}e^{-t} + \frac{13}{12}e^{2t} & \frac{5}{4}e^{-2t} - \frac{4}{3}e^{-t} + \frac{1}{12}e^{2t} \\ \frac{7}{2}e^{-2t} - 2e^{-t} - \frac{3}{2}e^{2t} & \frac{7}{4}e^{-2t} - \frac{2}{3}e^{-t} - \frac{13}{12}e^{2t} & \frac{7}{4}e^{-2t} - \frac{2}{3}e^{-t} - \frac{1}{12}e^{2t} \end{pmatrix}$
10. (b)  $\Phi(t) = \begin{pmatrix} \frac{1}{6}e^t + \frac{1}{3}e^{-2t} + \frac{1}{2}e^{3t} & -\frac{1}{3}e^t + \frac{1}{3}e^{-2t} & \frac{1}{2}e^t - e^{-2t} + \frac{1}{2}e^{3t} \\ -\frac{2}{3}e^t - \frac{1}{3}e^{-2t} + e^{3t} & \frac{4}{3}e^t - \frac{1}{3}e^{-2t} & -2e^t + e^{-2t} + e^{3t} \\ -\frac{1}{6}e^t - \frac{1}{3}e^{-2t} + \frac{1}{2}e^{3t} & \frac{1}{3}e^t - \frac{1}{3}e^{-2t} & -\frac{1}{2}e^t + e^{-2t} + \frac{1}{2}e^{3t} \end{pmatrix}$
11.  $\mathbf{x} = \frac{7}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t - \frac{3}{2} \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-t}$
12.  $\mathbf{x} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} e^{-t} \cos 2t + \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix} e^{-t} \sin 2t$
17. (c)  $\mathbf{x} = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} \cos \omega t + \begin{pmatrix} v_0 \\ -\omega^2 u_0 \end{pmatrix} \frac{\sin \omega t}{\omega}$

## Seção 7.8

1. (c)  $\mathbf{x} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^t + c_2 \left[ \begin{pmatrix} 2 \\ 1 \end{pmatrix} te^t + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t \right]$
2. (c)  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 \left[ \begin{pmatrix} 1 \\ 2 \end{pmatrix} t - \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix} \right]$
3. (c)  $\mathbf{x} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-t} + c_2 \left[ \begin{pmatrix} 2 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{-t} \right]$
4. (c)  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t/2} + c_2 \left[ \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t/2} + \begin{pmatrix} 0 \\ \frac{2}{5} \end{pmatrix} e^{-t/2} \right]$
5.  $\mathbf{x} = c_1 \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t} + c_3 \left[ \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} te^{2t} + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} e^{2t} \right]$
6.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} e^{-t} + c_3 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{-t}$
7. (a)  $\mathbf{x} = \begin{pmatrix} 3+4t \\ 2+4t \end{pmatrix} e^{-3t}$
8. (a)  $\mathbf{x} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} e^{-t} - 6 \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t}$
9. (a)  $\mathbf{x} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} e^{t/2} + \frac{3}{2} \begin{pmatrix} -1 \\ -1 \end{pmatrix} te^{t/2}$
10. (a)  $\mathbf{x} = 2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 14 \begin{pmatrix} 3 \\ -1 \end{pmatrix} t$
11. (a)  $\mathbf{x} = \begin{pmatrix} -1 \\ 2 \\ -33 \end{pmatrix} e^t + 4 \begin{pmatrix} 0 \\ 1 \\ -6 \end{pmatrix} te^t + 3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} e^{2t}$
12. (a)  $\mathbf{x} = \frac{4}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{-t/2} + \frac{1}{3} \begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix} e^{-7t/2}$
13.  $\mathbf{x} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} t + c_2 \left[ \begin{pmatrix} 2 \\ 1 \end{pmatrix} t \ln t + \begin{pmatrix} 1 \\ 0 \end{pmatrix} t \right]$
14.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} t^{-3} + c_2 \left[ \begin{pmatrix} 1 \\ 1 \end{pmatrix} t^{-3} \ln t - \begin{pmatrix} 0 \\ \frac{1}{4} \end{pmatrix} t^{-3} \right]$
16. (b)  $\begin{pmatrix} I \\ V \end{pmatrix} = - \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-t/2} + \left[ \begin{pmatrix} 1 \\ -2 \end{pmatrix} te^{-t/2} + \begin{pmatrix} 2 \\ 0 \end{pmatrix} e^{-t/2} \right]$

17. (b)  $\mathbf{x}^{(1)}(t) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t}$

(c)  $\mathbf{x}^{(2)}(t) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} te^{2t} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{2t}$

(d)  $\mathbf{x}^{(3)}(t) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} (t^2/2)e^{2t} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} te^{2t} + \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} e^{2t}$

(e)  $\Psi(t) = e^{2t} \begin{pmatrix} 0 & 1 & t+2 \\ 1 & t+1 & (t^2/2)+t \\ -1 & -t & -(t^2/2)+3 \end{pmatrix}$

(f)  $\mathbf{T} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ -1 & 0 & 3 \end{pmatrix}, \quad \mathbf{T}^{-1} = \begin{pmatrix} -3 & 3 & 2 \\ 3 & -2 & -2 \\ -1 & 1 & 1 \end{pmatrix}.$

$$\mathbf{J} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

18. (a)  $\mathbf{x}^{(1)}(t) = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} e^t, \quad \mathbf{x}^{(2)}(t) = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} e^t$

(d)  $\mathbf{x}^{(3)}(t) = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix} te^t + \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} e^t$

(e)  $\Psi(t) = e^t \begin{pmatrix} 1 & 0 & 2t \\ 0 & 2 & 4t \\ 2 & -3 & -2t-1 \end{pmatrix}$  ou  $e^t \begin{pmatrix} 1 & 2 & 2t \\ 0 & 4 & 4t \\ 2 & -2 & -2t-1 \end{pmatrix}$

(f)  $\mathbf{T} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 4 & 0 \\ 2 & -2 & -1 \end{pmatrix}, \quad \mathbf{T}^{-1} = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/4 & 0 \\ 2 & -3/2 & -1 \end{pmatrix}.$

$$\mathbf{J} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

19. (a)  $\mathbf{J}^2 = \begin{pmatrix} \lambda^2 & 2\lambda \\ 0 & \lambda^2 \end{pmatrix}, \quad \mathbf{J}^3 = \begin{pmatrix} \lambda^3 & 3\lambda^2 \\ 0 & \lambda^3 \end{pmatrix}, \quad \mathbf{J}^4 = \begin{pmatrix} \lambda^4 & 4\lambda^3 \\ 0 & \lambda^4 \end{pmatrix}$

(c)  $\exp(\mathbf{J}t) = e^{\lambda t} \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$

(d)  $\mathbf{x} = \exp(\mathbf{J}t)\mathbf{x}^0$

20. (c)  $\exp(\mathbf{J}t) = e^{\lambda t} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{pmatrix}$

21. (c)  $\exp(\mathbf{J}t) = e^{\lambda t} \begin{pmatrix} 1 & t & t^2/2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{pmatrix}$

### Seção 7.9

1.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-t} + \frac{3}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^t - \frac{1}{4} \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^t + \begin{pmatrix} 1 \\ 2 \end{pmatrix} t - \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

2.  $\mathbf{x} = c_1 \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} e^{-2t} - \begin{pmatrix} 2/3 \\ 1/\sqrt{3} \end{pmatrix} e^t + \begin{pmatrix} -1 \\ 2/\sqrt{3} \end{pmatrix} e^{-t}$

3.  $\mathbf{x} = c_1 \begin{pmatrix} 5 \cos t \\ 2 \cos t + \sin t \end{pmatrix} + c_2 \begin{pmatrix} 5 \sin t \\ -\cos t + 2 \sin t \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} t \cos t - \begin{pmatrix} 1 \\ 0 \end{pmatrix} t \sin t - \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cos t$

4.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-2t} + \frac{1}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t$

5.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 \left[ \begin{pmatrix} 1 \\ 2 \end{pmatrix} t - \frac{1}{2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] - 2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} \ln t + \begin{pmatrix} 2 \\ 5 \end{pmatrix} t^{-1} - \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix} t^{-2}$
6.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-5t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \ln t + \frac{8}{5} \begin{pmatrix} 1 \\ 2 \end{pmatrix} t + \frac{4}{25} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$
7.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-t} + \frac{1}{4} \begin{pmatrix} 1 \\ -8 \end{pmatrix} e^t$
8.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t + 2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} t e^t$
9.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t/2} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t} + \begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix} t - \begin{pmatrix} \frac{17}{4} \\ \frac{15}{4} \end{pmatrix} + \begin{pmatrix} \frac{1}{6} \\ \frac{1}{2} \end{pmatrix} e^t$
10.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} -\sqrt{2} \\ 1 \end{pmatrix} e^{-4t} - \frac{1}{3} \begin{pmatrix} \sqrt{2}-1 \\ 2-\sqrt{2} \end{pmatrix} t e^{-t} + \frac{1}{9} \begin{pmatrix} 2+\sqrt{2} \\ -1-\sqrt{2} \end{pmatrix} e^{-t}$
11.  $\mathbf{x} = c_1 \begin{pmatrix} 5 \cos t \\ 2 \cos t + \sin t \end{pmatrix} + c_2 \begin{pmatrix} 5 \sin t \\ -\cos t + 2 \sin t \end{pmatrix} + \begin{pmatrix} 0 \\ 1/2 \end{pmatrix} t \cos t - \begin{pmatrix} 5/2 \\ 1 \end{pmatrix} t \sin t - \begin{pmatrix} 5/2 \\ 1 \end{pmatrix} \cos t$
12.  $\mathbf{x} = [\frac{1}{5} \ln(\sin t) - \ln(-\cos t) - \frac{2}{5}t + c_1] \begin{pmatrix} 5 \cos t \\ 2 \cos t + \sin t \end{pmatrix} + [\frac{2}{5} \ln(\sin t) - \frac{4}{5}t + c_2] \begin{pmatrix} 5 \sin t \\ -\cos t + 2 \sin t \end{pmatrix}$
13. (a)  $\Psi(t) = \begin{pmatrix} e^{-t/2} \cos \frac{1}{2}t & e^{-t/2} \sin \frac{1}{2}t \\ 4e^{-t/2} \sin \frac{1}{2}t & -4e^{-t/2} \cos \frac{1}{2}t \end{pmatrix}$       (b)  $\mathbf{x} = e^{-t/2} \begin{pmatrix} \sin \frac{1}{2}t \\ 4 - 4 \cos \frac{1}{2}t \end{pmatrix}$
14.  $\mathbf{x} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} t + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} t^{-1} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 1 \\ 3 \end{pmatrix} t - \begin{pmatrix} 1 \\ 1 \end{pmatrix} t \ln t - \frac{1}{3} \begin{pmatrix} 4 \\ 3 \end{pmatrix} t^2$
15.  $\mathbf{x} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} t^2 + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} t^{-1} + \begin{pmatrix} 3 \\ 2 \end{pmatrix} t + \frac{1}{10} \begin{pmatrix} -2 \\ 1 \end{pmatrix} t^4 - \frac{1}{2} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

C A P Í T U L O 8

## Seção 8.1

1. (a) 1,1975; 1,38549; 1,56491; 1,73658  
 (b) 1,19631; 1,38335; 1,56200; 1,73308  
 (c) 1,19297; 1,37730; 1,55378; 1,72316  
 (d) 1,19405; 1,37925; 1,55644; 1,72638
2. (a) 1,59980; 1,29288; 1,07242; 0,930175  
 (b) 1,61124; 1,31361; 1,10012; 0,962552  
 (c) 1,64337; 1,37164; 1,17763; 1,05334  
 (d) 1,63301; 1,35295; 1,15267; 1,02407
3. (a) 1,2025; 1,41603; 1,64289; 1,88590  
 (b) 1,20388; 1,41936; 1,64896; 1,89572  
 (c) 1,20864; 1,43104; 1,67042; 1,93076  
 (d) 1,20693; 1,42683; 1,66265; 1,91802
4. (a) 1,10244; 1,21426; 1,33484; 1,46399  
 (b) 1,10365; 1,21656; 1,33817; 1,46832  
 (c) 1,10720; 1,22333; 1,34797; 1,48110  
 (d) 1,10603; 1,22110; 1,34473; 1,47688
5. (a) 0,509239; 0,522187; 0,539023; 0,559936  
 (b) 0,509701; 0,523155; 0,540550; 0,562089  
 (c) 0,511127; 0,526155; 0,545306; 0,568822  
 (d) 0,510645; 0,525138; 0,543690; 0,566529
6. (a) -0,920498; -0,857538; -0,808030; -0,770038  
 (b) -0,922575; -0,860923; -0,812300; -0,774965  
 (c) -0,928059; -0,870054; -0,824021; -0,788686  
 (d) -0,926341; -0,867163; -0,820279; -0,784275
7. (a) 2,90330; 7,53999; 19,4292; 50,5614  
 (b) 2,93506; 7,70957; 20,1081; 52,9779

- (c) 3,03951; 8,28137; 22,4562; 61,5496  
 (d) 3,00306; 8,07933; 21,6163; 58,4462
8. (a) 0,891830; 1,25225; 2,37818; 4,07257  
 (b) 0,908902; 1,26872; 2,39336; 4,08799  
 (c) 0,958565; 1,31786; 2,43924; 4,13474  
 (d) 0,942261; 1,30153; 2,42389; 4,11908
9. (a) 3,95713; 5,09853; 6,41548; 7,90174  
 (b) 3,95965; 5,10371; 6,42343; 7,91255  
 (c) 3,96727; 5,11932; 6,44737; 7,94512  
 (d) 3,96473; 5,11411; 6,43937; 7,93424
10. (a) 1,60729; 2,46830; 3,72167; 5,45963  
 (b) 1,60996; 2,47460; 3,73356; 5,47774  
 (c) 1,61792; 2,49356; 3,76940; 5,53223  
 (d) 1,61528; 2,48723; 3,75742; 5,51404
11. (a) -1,45865; -0,217545; 1,05715; 1,41487  
 (b) -1,45322; -0,180813; 1,05903; 1,41244  
 (c) -1,43600; -0,0681657; 1,06489; 1,40575  
 (d) -1,44190; -0,105737; 1,06290; 1,40789
12. (a) 0,587987; 0,791589; 1,14743; 1,70973  
 (b) 0,589440; 0,795758; 1,15693; 1,72955  
 (c) 0,593901; 0,808716; 1,18687; 1,79291  
 (d) 0,592396; 0,804319; 1,17664; 1,77111
15. 1,595; 2,4636
16.  $e_{n+1} = [2\phi(\bar{t}_n) - 1]h^2$ ,  $|e_{n+1}| \leq [1 + 2 \max_{0 \leq t \leq 1} |\phi(t)|] h^2$ ,  
 $e_{n+1} = e^{2\bar{t}_n} h^2$ ,  $|e_1| \leq 0,012$ ,  $|e_4| \leq 0,022$
17.  $e_{n+1} = [2\phi(\bar{t}_n) - \bar{t}_n]h^2$ ,  $|e_{n+1}| \leq [1 + 2 \max_{0 \leq t \leq 1} |\phi(t)|] h^2$ ,  
 $e_{n+1} = 2e^{2\bar{t}_n} h^2$ ,  $|e_1| \leq 0,024$ ,  $|e_4| \leq 0,045$
18.  $e_{n+1} = [\bar{t}_n + \bar{t}_n^2 \phi(\bar{t}_n) + \phi^3(\bar{t}_n)]h^2$       19.  $e_{n+1} = [19 - 15\bar{t}_n \phi^{-1/2}(\bar{t}_n)]h^2/4$
20.  $e_{n+1} = \{1 + [\bar{t}_n + \phi(\bar{t}_n)]^{-1/2}\}h^2/4$
21.  $e_{n+1} = \{2 - [\phi(\bar{t}_n) + 2\bar{t}_n^2] \exp[-\bar{t}_n \phi(\bar{t}_n)] - \bar{t}_n \exp[-2\bar{t}_n \phi(\bar{t}_n)]\}h^2/2$
22. (a)  $\phi(t) = 1 + (1/5\pi) \operatorname{sen} 5\pi t$       (b) 1,2; 1,0; 1,2  
 (c) 1,1; 1,1; 1,0; 1,0      (d)  $h < 1/\sqrt{50\pi} \cong 0,08$
24.  $e_{n+1} = -\frac{1}{2}\phi''(\bar{t}_n)h^2$
25. (a) 1,55; 2,34; 3,46; 5,07  
 (b) 1,20; 1,39; 1,57; 1,74  
 (c) 1,20; 1,42; 1,65; 1,90
26. (a) 0      (b) 60      (c) -92,16      27. 0,224 ≠ 0,225

## Seção 8.2

1. (a) 1,19512; 1,38120; 1,55909; 1,72956  
 (b) 1,19515; 1,38125; 1,55916; 1,72965  
 (c) 1,19516; 1,38126; 1,55918; 1,72967
2. (a) 1,62283; 1,33460; 1,12820; 0,995445  
 (b) 1,62243; 1,33386; 1,12718; 0,994215  
 (c) 1,62234; 1,33368; 1,12693; 0,993921
3. (a) 1,20526; 1,42273; 1,65511; 1,90570  
 (b) 1,20533; 1,42290; 1,65542; 1,90621  
 (c) 1,20534; 1,42294; 1,65550; 1,90634
4. (a) 1,10483; 1,21882; 1,34146; 1,47263  
 (b) 1,10484; 1,21884; 1,34147; 1,47262  
 (c) 1,10484; 1,21884; 1,34147; 1,47262
5. (a) 0,510164; 0,524126; 0,542083; 0,564251  
 (b) 0,510168; 0,524135; 0,542100; 0,564277  
 (c) 0,510169; 0,524137; 0,542104; 0,564284
6. (a) -0,924650; -0,864338; -0,816642; -0,780008  
 (b) -0,924550; -0,864177; -0,816442; -0,779781  
 (c) -0,924525; -0,864138; -0,816393; -0,779725
7. (a) 2,96719; 7,88313; 20,8114; 55,5106  
 (b) 2,96800; 7,88755; 20,8294; 55,5758

### Seção 8.3

- (a) 1,19516; 1,38127; 1,55918; 1,72968  
 (b) 1,19516; 1,38127; 1,55918; 1,72968
  - (a) 1,62231; 1,33362; 1,12686; 0,993839  
 (b) 1,62230; 1,33362; 1,12685; 0,993826
  - (a) 1,20535; 1,42295; 1,65553; 1,90638  
 (b) 1,20535; 1,42296; 1,65553; 1,90638
  - (a) 1,10484; 1,21884; 1,34147; 1,47262  
 (b) 1,10484; 1,21884; 1,34147; 1,47262
  - (a) 0,510170; 0,524138; 0,542105; 0,564286  
 (b) 0,520169; 0,524138; 0,542105; 0,564286
  - (a) -0,924517; -0,864125; -0,816377; -0,779706  
 (b) -0,924517; -0,864125; -0,816377; -0,779706
  - (a) 2,96825; 7,88889; 20,8349; 55,5957  
 (b) 2,96828; 7,88904; 20,8355; 55,5980
  - (a) 0,925725; 1,28516; 2,40860; 4,10350  
 (b) 0,925711; 1,28515; 2,40860; 4,10350
  - (a) 3,96219; 5,10890; 6,43139; 7,92338  
 (b) 3,96219; 5,10890; 6,43139; 7,92338
  - (a) 1,61262; 2,48091; 3,74548; 5,49587  
 (b) 1,61262; 2,48091; 3,74548; 5,49587
  - (a) -1,44764; -0,143543; 1,06089; 1,41008  
 (b) -1,44764; -0,143427; 1,06095; 1,41011
  - (a) 0,590909; 0,800000; 1,166667; 1,75000  
 (b) 0,590909; 0,800000; 1,166667; 1,75000

## Seção 8.4

- (a) 1,7296801; 1.8934697  
 (b) 1,7296802; 1.8934698  
 (c) 1,7296805; 1.8934711
  - (a) 0,993852; 0,925764  
 (b) 0,993846; 0,925746  
 (c) 0,993869; 0,925837
  - (a) 1,906382; 2,179567  
 (b) 1,906391; 2,179582  
 (c) 1,906395; 2,179611
  - (a) 1,4726173; 1,6126215  
 (b) 1,4726189; 1,6126231  
 (c) 1,4726199; 1,6126256

5. (a) 0.56428577; 0.59090918  
     (b) 0.56428581; 0.59090923  
     (c) 0.56428588; 0.59090952

6. (a) -0.779693; -0.753135  
     (b) -0.779692; -0.753137  
     (c) -0.779680; -0.753089

7. (a) 2.96828; 7.88907; 20.8356; 55.5984  
     (b) 2.96829; 7.88909; 20.8357; 55.5986  
     (c) 2.96831; 7.88926; 20.8364; 55.6015

8. (a) 0.9257133; 1.285148; 2.408595; 4.103495  
     (b) 0.9257124; 1.285148; 2.408595; 4.103495  
     (c) 0.9257248; 1.285158; 2.408594; 4.103493

9. (a) 3.962186; 5.108903; 6.431390; 7.923385  
     (b) 3.962186; 5.108903; 6.431390; 7.923385  
     (c) 3.962186; 5.108903; 6.431390; 7.923385

10. (a) 1.612622; 2.480909; 3.745479; 5.495872  
     (b) 1.612622; 2.480909; 3.745479; 5.495873  
     (c) 1.612623; 2.480905; 3.745473; 5.495869

11. (a) -1.447639; -0.1436281; 1.060946; 1.410122  
     (b) -1.447638; -0.1436762; 1.060913; 1.410103  
     (c) -1.447621; -0.1447219; 1.060717; 1.410027

12. (a) 0.5909091; 0.8000000; 1.166667; 1,750000  
     (b) 0.5909091; 0.8000000; 1.166667; 1,750000  
     (c) 0.5909092; 0.8000002; 1.166667; 1,750001

## Seção 8.5

- (b)  $\phi_2(t) - \phi_1(t) = 0,001e^t \rightarrow \infty$  quando  $t \rightarrow \infty$
  - (b)  $\phi_1(t) = \ln[e^t/(2-e^t)]; \quad \phi_2(t) = \ln[1/(1-t)]$
  - (a,b)  $h = 0,00025$  é suficiente. (c)  $h = 0,005$  é suficiente.
  - (a)  $y = 4e^{-10t} + (t^2/4)$ . (c) O método de Runge-Kutta é estável para  $h = 0,25$ , mas é instável para  $h = 0,3$ .  
(d)  $h = 5/13 \approx 0,384615$  é suficientemente pequeno.
  - (a)  $y = t$
  - (a)  $y = t^2$

## Seção 8.6

- (a) 1.26, 0.76; 1.7714, 1.4824; 2.58991, 2.3703; 3.82374, 3.60413;  
5,64246, 5,38885  
(b) 1.32493, 0.758933; 1.93679, 1.57919; 2,93414, 2,66099; 4,48318, 4.22639;  
6,84236, 6,56452  
(c) 1.32489, 0.759516; 1.9369, 1.57999; 2,93459, 2,66201; 4,48422, 4.22784;  
6,8444, 6,56684
  - (a) 1.451, 1.232; 2,16133, 1.65988; 3,29292, 2,55559; 5,16361, 4,7916;  
8,54951, 12,0464  
(b) 1,51844, 1.28089; 2,37684, 1.87711; 3,85039, 3,44859; 6,6956, 9,50309;  
15,0987, 64,074  
(c) 1,51855, 1,2809; 2,3773, 1,87729; 3,85247, 3,45126; 6,71282, 9,56846;  
15,6384, 70,3792
  - (a) 0.582, 1.18; 0.117969, 1,27344; -0,336912, 1,27382; -0,730007, 1.18572;  
-1,02134, 1,02371  
(b) 0.568451, 1,15775; 0.109776, 1.22556; -0,32208, 1,20347;  
-0,681296, 1,10162; -0.937852, 0.937852  
(c) 0.56845, 1,15775; 0,109773, 1.22557; -0,322081, 1,20347;  
-0,681291, 1,10161; -0.937841, 0.93784
  - (a) -0,198, 0,618; -0,378796, 0.28329; -0,51932, -0,0321025;  
-0,594324, -0,326801; -0,588278, -0,57545  
(b) -0,196904, 0,630936; -0,372643, 0,298888; -0,501302, -0,0111429;  
-0,561270, -0,288943; -0,547053, -0,508303  
(c) -0,196935, 0,630939; -0,372687, 0,298866; -0,501345, -0,0112184;  
-0,561292, -0,28907; -0,547031, -0,508427

5. (a) 2,96225, 1,34538; 2,34119, 1,67121; 1,90236, 1,97158; 1,56602, 2,23895;  
1,29768, 2,46732  
(b) 3,06339, 1,34858; 2,44497, 1,68638; 1,9911, 2,00036; 1,63818, 2,27981;  
1,3555, 2,5175  
(c) 3,06314, 1,34899; 2,44465, 1,68699; 1,99075, 2,00107; 1,63781, 2,28057;  
1,35514, 2,51827
6. (a) 1,42386, 2,18957; 1,82234, 2,36791; 2,21728, 2,53329; 2,61118, 2,68763;  
2,9955, 2,83354  
(b) 1,41513, 2,18699; 1,81208, 2,36233; 2,20635, 2,5258; 2,59826, 2,6794;  
2,97806, 2,82487  
(c) 1,41513, 2,18699; 1,81209, 2,36233; 2,20635, 2,52581; 2,59826, 2,67941;  
2,97806, 2,82488
7. Para  $h = 0,05$  e  $0,025$ :  $x = 10,227$ ,  $y = -4,9294$ ; estes resultados estão de acordo com a solução exata até cinco dígitos
8. 1,543, 0,0707503; 1,14743, -1,3885      9. 1,99521, -0,662442

**CAPÍTULO 9****Seção 9.1**

1. (a)  $r_1 = -1$ ,  $\xi^{(1)} = (1, 2)^T$ ;  $r_2 = 2$ ,  $\xi^{(2)} = (2, 1)^T$  (b) ponto de sela, instável
2. (a)  $r_1 = 2$ ,  $\xi^{(1)} = (1, 3)^T$ ;  $r_2 = 4$ ,  $\xi^{(2)} = (1, 1)^T$  (b) nó, instável
3. (a)  $r_1 = -1$ ,  $\xi^{(1)} = (1, 3)^T$ ;  $r_2 = 1$ ,  $\xi^{(2)} = (1, 1)^T$  (b) ponto de sela, instável
4. (a)  $r_1 = r_2 = -3$ ;  $\xi^{(1)} = (1, 1)^T$  (b) nó impróprio, assintoticamente estável
5. (a)  $r_1, r_2 = -1 \pm i$ ;  $\xi^{(1)}, \xi^{(2)} = (2 \pm i, 1)^T$  (b) ponto espiral, assintoticamente estável
6. (a)  $r_1, r_2 = \pm i$ ;  $\xi^{(1)}, \xi^{(2)} = (2 \pm i, 1)^T$  (b) centro, estável
7. (a)  $r_1, r_2 = 1 \pm 2i$ ;  $\xi^{(1)}, \xi^{(2)} = (1, 1 \mp i)^T$  (b) ponto espiral, instável
8. (a)  $r_1 = -1$ ,  $\xi^{(1)} = (1, 0)^T$ ;  $r_2 = -1/4$ ,  $\xi^{(2)} = (4, -3)^T$  (b) nó, assintoticamente estável
9. (a)  $r_1 = r_2 = 1$ ;  $\xi^{(1)} = (2, 1)^T$  (b) nó impróprio, instável
10. (a)  $r_1, r_2 = \pm 3i$ ;  $\xi^{(1)}, \xi^{(2)} = (2, -1 \pm 3i)^T$  (b) centro, estável
11. (a)  $r_1 = r_2 = -1$ ;  $\xi^{(1)} = (1, 0)^T$ ,  $\xi^{(2)} = (0, 1)^T$  (b) nó próprio, assintoticamente estável
12. (a)  $r_1, r_2 = (1 \pm 3i)/2$ ;  $\xi^{(1)}, \xi^{(2)} = (5, 3 \mp 3i)^T$  (b) ponto espiral, instável
13.  $x_0 = 1$ ,  $y_0 = 1$ ;  $r_1 = \sqrt{2}$ ,  $r_2 = -\sqrt{2}$ ; ponto de sela, instável
14.  $x_0 = -1$ ,  $y_0 = 0$ ;  $r_1 = -1$ ,  $r_2 = -3$ ; nó, assintoticamente estável
15.  $x_0 = -2$ ,  $y_0 = 1$ ;  $r_1, r_2 = -1 \pm \sqrt{2}i$ ; ponto espiral, assintoticamente estável
16.  $x_0 = \gamma/\delta$ ,  $y_0 = \alpha/\beta$ ;  $r_1, r_2 = \pm \sqrt{\beta\delta}i$ ; centro, estável
17.  $c^2 > 4km$ , nó, assintoticamente estável;  $c^2 = 4km$ , nó impróprio, assintoticamente estável;  $c^2 < 4km$ , ponto espiral, assintoticamente estável

**Seção 9.2**

1.  $x = 4e^{-t}$ ,  $y = 2e^{-2t}$ ,  $y = x^2/8$
2.  $x = 4e^{-t}$ ,  $y = 2e^{2t}$ ,  $y = 32x^{-2}$ ;  $x = 4e^{-t}$ ,  $y = 0$
3.  $x = 4 \cos t$ ,  $y = 4 \sin t$ ,  $x^2 + y^2 = 16$ ;  $x = -4 \sin t$ ,  $y = 4 \cos t$ ,  $x^2 + y^2 = 16$
4.  $x = \sqrt{a} \cos \sqrt{ab}t$ ,  $y = -\sqrt{b} \sin \sqrt{ab}t$ ;  $(x^2/a) + (y^2/b) = 1$
5. (a, c)  $(-\frac{1}{2}, 1)$ , ponto de sela, instável;  $(0, 0)$ , nó (próprio), instável
6. (a, c)  $(-\sqrt{3}/3, -\frac{1}{2})$ , ponto de sela, instável;  $(\sqrt{3}/3, -\frac{1}{2})$ , centro, estável
7. (a, c)  $(0, 0)$ , nó, instável;  $(2, 0)$ , nó, assintoticamente estável;  $(0, \frac{3}{2})$ , ponto de sela, instável;  $(-1, 3)$ , nó, assintoticamente estável
8. (a, c)  $(0, 0)$ , nó, assintoticamente estável;  $(1, -1)$ , ponto de sela, instável;  $(1, -2)$ , ponto aspiral, assintoticamente estável
9. (a, c)  $(0, 0)$ , ponto espiral, assintoticamente estável;  $(1 - \sqrt{2}, 1 + \sqrt{2})$ , ponto de sela, instável;  $(1 + \sqrt{2}, 1 - \sqrt{2})$ , ponto de sela, instável
10. (a, c)  $(0, 0)$ , ponto de sela, instável;  $(2, 2)$ , ponto espiral, assintoticamente estável;  $(-1, -1)$ , ponto espiral, assintoticamente estável;  $(-2, 0)$ , ponto de sela, instável
11. (a, c)  $(0, 0)$ , ponto de sela, instável;  $(0, 1)$ , ponto de sela, instável;  $(\frac{1}{2}, \frac{1}{2})$ , centro, estável;  $(-\frac{1}{2}, \frac{1}{2})$ , centro, estável
12. (a, c)  $(0, 0)$ , ponto de sela, instável;  $(\sqrt{6}, 0)$ , ponto espiral, assintoticamente estável;  $(-\sqrt{6}, 0)$ , ponto espiral, assintoticamente estável
13. (a, c)  $(0, 0)$ , ponto de sela, instável;  $(-2, 2)$ , nó, instável;  $(4, 4)$ , ponto espiral, assintoticamente estável
14. (a, c)  $(0, 0)$ , ponto de sela, instável;  $(2, 0)$ , ponto de sela, instável;  $(1, 1)$ , ponto espiral, assintoticamente estável;  $(-2, -2)$ , ponto espiral, assintoticamente estável
15. (a, c)  $(0, 0)$ , nó, instável;  $(1, 1)$ , ponto de sela, instável;  $(3, -1)$ , ponto espiral, assintoticamente estável

16. (a, c)  $(0, 1)$ , ponto de sela, instável;  $(1, 1)$ , nó, assintoticamente estável;  $(-2, 4)$ , ponto espiral, instável  
 17. (a)  $4x^2 - y^2 = c$       18. (a)  $4x^2 + y^2 = c$   
 19. (a)  $(y - 2x)^2(x + y) = c$       20. (a)  $\arctan(y/x) - \ln\sqrt{x^2 + y^2} = c$   
 21. (a)  $2x^2y - 2xy + y^2 = c$       22. (a)  $x^2y^2 - 3x^2y - 2y^2 = c$   
 23. (a)  $(y^2/2) - \cos x = c$       24. (a)  $x^2 + y^2 - (x^4/12) = c$

### Seção 9.3

1. Linear e não linear: ponto de sela, instável
2. Linear e não linear: ponto espiral, assintoticamente estável
3. Linear: centro, estável; não linear: ponto espiral ou centro, indeterminado
4. Linear: nó impróprio, instável; não linear: nó ou ponto espiral, instável
5. (a, b, c)  $(0, 0)$ ;  $u' = -2u + 2v$ ,  $v' = 4u + 4v$ ;  $r = 1 \pm \sqrt{17}$ ; ponto de sela, instável  
 $(-2, 2)$ ;  $u' = 4u$ ,  $v' = 6u + 6v$ ;  $r = 4, 6$ ; nó, instável  
 $(4, 4)$ ;  $u' = -6u + 6v$ ,  $v' = -8u$ ;  $r = -3 \pm \sqrt{39}i$ ; ponto espiral, assintoticamente estável
6. (a, b, c)  $(0, 0)$ ;  $u' = u$ ,  $v' = 3v$ ;  $r = 1, 3$ ; nó, instável  
 $(1, 0)$ ;  $u' = -u - v$ ,  $v' = 2v$ ;  $r = -1, 2$ ; ponto de sela, instável  
 $(0, \frac{3}{2})$ ;  $u' = (-\frac{1}{2})u$ ,  $v' = (-\frac{3}{2})u - 3v$ ;  $r = -\frac{1}{2}, -3$ ; nó, assintoticamente estável  
 $(-1, 2)$ ;  $u' = u + v$ ,  $v' = -2u - 4v$ ;  $r = (-3 \pm \sqrt{17})/2$ ; ponto de sela, instável
7. (a, b, c)  $(1, 1)$ ;  $u' = -v$ ,  $v' = 2u - 2v$ ;  $r = -1 \pm i$ ; ponto espiral, assintoticamente estável  
 $(-1, 1)$ ;  $u' = -v$ ,  $v' = -2u - 2v$ ;  $r = -1 \pm \sqrt{3}$ ; ponto de sela, instável
8. (a, b, c)  $(0, 0)$ ;  $u' = u$ ,  $v' = (\frac{1}{2})v$ ;  $r = 1, \frac{1}{2}$ ; nó, instável  
 $(0, 2)$ ;  $u' = -u$ ,  $v' = (-\frac{3}{2})u - (\frac{1}{2})v$ ;  $r = -1, -\frac{1}{2}$ ; nó, assintoticamente estável  
 $(1, 0)$ ;  $u' = -u - v$ ,  $v' = (-\frac{1}{4})v$ ;  $r = -1, -1/4$ ; nó, assintoticamente estável  
 $(\frac{1}{2}, \frac{1}{2})$ ;  $u' = (-\frac{1}{2})u - (\frac{1}{2})v$ ,  $v' = (-\frac{3}{8})u - (\frac{1}{8})v$ ;  $r = (-5 \pm \sqrt{57})/16$ ; ponto de sela, instável
9. (a, b, c)  $(0, 0)$ ;  $u' = -u + 2v$ ,  $v' = u + 2v$ ;  $r = (1 \pm \sqrt{17})/2$ ; ponto de sela, instável  
 $(2, 1)$ ;  $u' = (-\frac{3}{2})u + 3v$ ,  $v' = -2u$ ;  $r = (-3 \pm \sqrt{87}i)/4$ ; ponto espiral, assintoticamente estável  
 $(2, -2)$ ;  $u' = -3v$ ,  $v' = u$ ;  $r = \pm\sqrt{3}i$ ; centro ou ponto espiral, indeterminado  
 $(4, -2)$ ;  $u' = -4v$ ,  $v' = -u - 2v$ ;  $r = -1 \pm \sqrt{5}$ ; ponto de sela, instável
10. (a, b, c)  $(0, 0)$ ;  $u' = u$ ,  $v' = v$ ;  $r = 1, 1$ ; nó ou ponto espiral, instável  
 $(-1, 0)$ ;  $u' = -u$ ,  $v' = 2v$ ;  $r = -1, 2$ ; ponto de sela, instável
11. (a, b, c)  $(0, 0)$ ;  $u' = 2u + v$ ,  $v' = u - 2v$ ;  $r = \pm\sqrt{5}$ ; ponto de sela, instável  $(-1, 1935; -1, 4797); u' = -1, 2399u - 6, 8393v$ ,  $v' = 2, 4797u - 0, 80655v$ ;  $r = -1, 0232 \pm 4, 1125i$ ; ponto espiral, assintoticamente estável
12. (a, b, c)  $(0, \pm 2n\pi)$ ,  $n = 0, 1, 2, \dots$ ;  $u' = v$ ,  $v' = -u$ ;  $r = \pm i$ ; centro ou ponto espiral, indeterminado  
 $(2, \pm 2(n-1)\pi)$ ,  $n = 1, 2, 3, \dots$ ;  $u' = -3v$ ,  $v' = -u$ ;  $r = \pm\sqrt{3}$ ; ponto de sela, instável
13. (a, b, c)  $(0, 0)$ ;  $u' = u$ ,  $v' = v$ ;  $r = 1, 1$ ; nó ou ponto espiral, instável  
 $(1, 1)$ ;  $u' = u - 2v$ ,  $v' = -2u + v$ ;  $r = 3, -1$ ; ponto de sela, instável
14. (a, b, c)  $(1, 1)$ ;  $u' = -u - v$ ,  $v' = u - 3v$ ;  $r = -2, -2$ ; nó ou ponto espiral, assintoticamente estável  
 $(-1, -1)$ ;  $u' = u + v$ ,  $v' = u - 3v$ ;  $r = -1 \pm \sqrt{5}$ ; ponto de sela, instável
15. (a, b, c)  $(0, 0)$ ;  $u' = -2u - v$ ,  $v' = u - v$ ;  $r = (-3 \pm \sqrt{3}i)/2$ ; ponto espiral, assintoticamente estável  
 $(-0, 33076; 1, 0924)$  e  $(0, 33076; -1, 0924)$ ;  $u' = -3, 5216u - 0, 27735v$ ,  $v' = 0, 27735u + 2, 6895v$ ;  $r = -3, 5092; 2, 6771$ ; ponto de sela, instável
16. (a, b, c)  $(0, 0)$ ;  $u' = u + v$ ,  $v' = -u + v$ ;  $r = 1 \pm i$ ; ponto espiral, instável
17. (a, b, c)  $(2, 2)$ ;  $u' = -4v$ ,  $v' = (-\frac{7}{2})u + (\frac{7}{2})v$ ;  $r = (7 \pm \sqrt{273})/4$ ; ponto de sela, instável  
 $(-2, -2)$ ;  $u' = 4v$ ,  $v' = (\frac{1}{2})u - (\frac{1}{2})v$ ;  $r = (-1 \pm \sqrt{33})/4$ ; ponto de sela, instável  
 $(-\frac{3}{2}, 2)$ ;  $u' = -4v$ ,  $v' = (\frac{7}{2})u$ ;  $r = \pm\sqrt{14}i$ ; centro ou ponto espiral, indeterminado  
 $(-\frac{3}{2}, -2)$ ;  $u' = 4v$ ,  $v' = (-\frac{1}{2})u$ ;  $r = \pm\sqrt{2}i$ ; centro ou ponto espiral, indeterminado
18. (a, b, c)  $(0, 0)$ ;  $u' = 2u - v$ ,  $v' = 2u - 4v$ ;  $r = -1 \pm \sqrt{7}$ ; ponto de sela, instável  
 $(2, 1)$ ;  $u' = -3v$ ,  $v' = 4u - 8v$ ;  $r = -2, -6$ ; nó, assintoticamente estável  
 $(-2, 1)$ ;  $u' = 5v$ ,  $v' = -4u$ ;  $r = \pm 2\sqrt{5}i$ ; centro ou ponto espiral, indeterminado  
 $(-2, -4)$ ;  $u' = 10u - 5v$ ,  $v' = 6u$ ;  $r = 5 \pm \sqrt{5}i$ ; ponto espiral, instável
21. (b, c) Veja a Tabela 9.3.1
23. (a)  $R = A$ ,  $T \cong 3,17$       (b)  $R = A$ ,  $T \cong 3,20; 3,35; 3,63; 4,17$   
 (c)  $T \rightarrow \pi$  quando  $A \rightarrow 0$       (d)  $A = \pi$
24. (b)  $v_c \cong 4,00$
25. (b)  $v_c \cong 4,51$
30. (a)  $dx/dt = y$ ,  $dy/dt = -g(x) - c(x)y$   
 (b) O sistema linear é  $dx/dt = y$ ,  $dy/dt = -g'(0)x - c(0)y$   
 (c) Os autovalores satisfazem  $r^2 + c(0)r + g'(0) = 0$

## Seção 9.4

1. (b, c)  $(0, 0); u' = (\frac{3}{2})u, v' = 2v; r = \frac{3}{2}, 2$ ; nó, instável  
 $(0, 2); u' = (\frac{1}{2})u, v' = (-\frac{3}{2})u - 2v; r = \frac{1}{2}, -2$ ; ponto de sela, instável  
 $(\frac{3}{2}, 0); u' = (-\frac{3}{2})u - (\frac{3}{4})v, v' = (\frac{7}{8})v; r = -\frac{3}{2}, \frac{7}{8}$ ; ponto de sela, instável  
 $(\frac{4}{5}, \frac{7}{5}); u' = (-\frac{4}{5})u - (\frac{2}{5})v, v' = (-\frac{21}{10})u - (\frac{7}{5})v; r = (-22 \pm \sqrt{204})/20$ ; nó, assintoticamente estável
2. (b, c)  $(0, 0); u' = (\frac{3}{2})u, v' = 2v; r = \frac{3}{2}, 2$ ; nó, instável  
 $(0, 4); u' = (-\frac{1}{2})u, v' = -6u - 2v; r = -\frac{1}{2}, -2$ ; nó, assintoticamente estável  
 $(\frac{3}{2}, 0); u' = (-\frac{3}{2})u - (\frac{3}{4})v, v' = (-\frac{1}{4})v; r = -\frac{3}{4}, -\frac{3}{2}$ ; nó, assintoticamente estável  
 $(1, 1); u' = -u - (\frac{1}{2})v, v' = (-\frac{3}{2})u - (\frac{1}{2})v; r = (-3 \pm \sqrt{13})/4$ ; ponto de sela, instável
3. (b, c)  $(0, 0); u' = (\frac{3}{2})u, v' = 2v; r = \frac{3}{2}, 2$ ; nó, instável  
 $(0, 2); u' = (-\frac{1}{2})u, v' = (-\frac{9}{4})u - 2v; r = -\frac{1}{2}, -2$ ; nó, assintoticamente estável  
 $(3, 0); u' = (-\frac{3}{2})u - 3v, v' = (-\frac{11}{8})v; r = -\frac{3}{2}, -\frac{11}{8}$ ; nó, assintoticamente estável  
 $(\frac{4}{5}, \frac{11}{10}); u' = (-\frac{2}{5})u - (\frac{4}{5})v, v' = (-\frac{99}{80})u - (\frac{11}{10})v; r = -1,80475; 0,30475$ ; ponto de sela, instável
4. (b, c)  $(0, 0); u' = (\frac{3}{2})u, v' = (\frac{3}{4})v; r = \frac{3}{2}, \frac{3}{4}$ ; nó, instável  
 $(0, \frac{3}{4}); u' = (\frac{3}{4})u, v' = (-\frac{3}{4})v; r = \pm \frac{3}{4}$ ; ponto de sela, instável  
 $(3, 0); u' = (-\frac{3}{2})u - 3v, v' = (\frac{3}{8})v; r = -\frac{3}{2}, \frac{3}{8}$ ; ponto de sela, instável  
 $(2, \frac{1}{2}); u' = -u - 2v, v' = (-\frac{1}{16})u - (\frac{1}{2})v; r = -1,18301; -0,31699$ ; nó, assintoticamente estável
5. (b, c)  $(0, 0); u' = u, v' = (\frac{3}{2})v; r = 1, \frac{3}{2}$ ; nó, instável  
 $(0, \frac{3}{2}); u' = (-\frac{1}{2})u, v' = (-\frac{3}{2})u - (\frac{3}{2})v; r = -\frac{1}{2}, -\frac{3}{2}$ ; nó, assintoticamente estável  
 $(1, 0); u' = -u - v, v' = (\frac{1}{2})v; r = -1, \frac{1}{2}$ ; ponto de sela, instável
6. (b, c)  $(0, 0); u' = u, v' = (\frac{5}{2})v; r = 1, \frac{5}{2}$ ; nó, instável  
 $(0, \frac{5}{3}); u' = (\frac{11}{6})u, v' = (\frac{5}{12})u - (\frac{5}{2})v; r = \frac{11}{6}, -5/2$ ; ponto de sela, instável  
 $(1, 0); u' = -u + (\frac{1}{2})v, v' = (\frac{11}{4})v; r = -1, \frac{11}{4}$ ; ponto de sela, instável  
 $(2, 2); u' = -2u + v, v' = (\frac{1}{2})u - 3v; r = (-5 \pm \sqrt{3})/2$ ; nó, assintoticamente estável
8. (a) Os pontos críticos são  $x = 0, y = 0; x = \epsilon_1/\sigma_1, y = 0; x = 0, y = \epsilon_2/\sigma_2$ .  $x \rightarrow 0, y \rightarrow \epsilon_2/\sigma_2$  quando  $t \rightarrow \infty$ ; os vermelhões sobrevivem  
(b) Os mesmos pontos críticos que em (a), mas  $x \rightarrow \epsilon_1/\sigma_1, y \rightarrow 0$  quando  $t \rightarrow \infty$ ; os peixes azulados sobrevivem
9. (a)  $X = (B - \gamma_1 R)/(1 - \gamma_1 \gamma_2), Y = (R - \gamma_2 B)/(1 - \gamma_1 \gamma_2)$   
(b)  $X$  diminui,  $Y$  aumenta; sim, se  $B$  se tornar menor do que  $\gamma_1 R$ , então  $x \rightarrow 0$  e  $y \rightarrow R$  quando  $t \rightarrow \infty$
10. (a)  $\sigma_1 \epsilon_2 - \alpha_2 \epsilon_1 \neq 0$ :  $(0, 0), (0, \epsilon_2/\sigma_2), (\epsilon_1/\sigma_1, 0)$   
 $\sigma_1 \epsilon_2 - \alpha_2 \epsilon_1 = 0$ :  $(0, 0)$  e todos os pontos na reta  $\sigma_1 x + \alpha_1 y = \epsilon_1$   
(b)  $\sigma_1 \epsilon_2 - \alpha_2 \epsilon_1 > 0$ :  $(0, 0)$  é um nó instável;  $(\epsilon_1/\sigma_1, 0)$  é um ponto de sela;  $(0, \epsilon_2/\sigma_2)$  é um nó assintoticamente estável  
 $\sigma_1 \epsilon_2 - \alpha_2 \epsilon_1 < 0$ :  $(0, 0)$  é um nó instável;  $(0, \epsilon_2/\sigma_2)$  é um ponto de sela;  $(\epsilon_1/\sigma_1, 0)$  é um nó assintoticamente estável  
(c)  $(0, 0)$  é um nó instável; os pontos na reta  $\sigma_1 x + \alpha_1 y = \epsilon_1$  são pontos críticos estáveis, não isolados
12. (a)  $(0, 0)$ , ponto de sela;  $(0, 15/0)$ , ponto espiral se  $\gamma^2 < 1,11$ ;  $(2, 0)$ , ponto de sela  
(c)  $\gamma \cong 1,20$
13. (b)  $(2 - \sqrt{4 - \frac{3}{2}\alpha}, \frac{3}{2}\alpha), (2 + \sqrt{4 - \frac{3}{2}\alpha}, \frac{3}{2}\alpha)$   
(c)  $(1, 3)$  é um nó assintoticamente estável;  $(3, 3)$  é um ponto de sela  
(d)  $\alpha_0 = 8/3$ ; o ponto crítico é  $(2, 4)$ ;  $\lambda = 0, -1$
14. (b)  $(2 - \sqrt{4 - \frac{3}{2}\alpha}, \frac{3}{2}\alpha), (2 + \sqrt{4 - \frac{3}{2}\alpha}, \frac{3}{2}\alpha)$   
(c)  $(1, 3)$  é um ponto de sela;  $(3, 3)$  é um ponto espiral instável  
(d)  $\alpha_0 = 8/3$ ; o ponto crítico é  $(2, 4)$ ;  $\lambda = 0, 1$
15. (b)  $([3 - \sqrt{9 - 4\alpha}]/2, [3 + 2\alpha - \sqrt{9 - 4\alpha}]/2), ([3 + \sqrt{9 - 4\alpha}]/2, [3 + 2\alpha + \sqrt{9 - 4\alpha}]/2)$   
(c)  $(1, 3)$  é um ponto de sela;  $(2, 4)$  é um ponto espiral instável  
(d)  $\alpha_0 = 9/4$ ; o ponto crítico é  $(3/2, 15/4)$ ;  $\lambda = 0, 0$
16. (b)  $([3 - \sqrt{9 - 4\alpha}]/2, [3 + 2\alpha - \sqrt{9 - 4\alpha}]/2), ([3 + \sqrt{9 - 4\alpha}]/2, [3 + 2\alpha + \sqrt{9 - 4\alpha}]/2)$   
(c)  $(1, 3)$  é um centro da aproximação linear e também do sistema linear;  $(2, 4)$  é um ponto de sela  
(d)  $\alpha_0 = 9/4$ ; o ponto crítico é  $(3/2, 15/4)$ ;  $\lambda = 0, 0$
17. (b)  $P_1(0, 0), P_2(1, 0), P_3(0, \alpha), P_4(2 - 2\alpha, -1 + 2\alpha)$ .  $P_4$  está no primeiro quadrante para  $0,5 \leq \alpha \leq 1$ .  
(c)  $\alpha = 0$ ;  $P_3$  coincide com  $P_1$ .  $\alpha = 0,5$ ;  $P_4$  coincide com  $P_2$ .  $\alpha = 1$ ;  $P_4$  coincide com  $P_3$ .  
(d)  $\mathbf{J} = \begin{pmatrix} 1 - 2x - y & -x \\ -0,5y & \alpha - 2y - 0,5x \end{pmatrix}$

- (e)  $P_1$  é um nó instável para  $\alpha > 0$ .  $P_2$  é um nó assintoticamente estável para  $0 < \alpha < 0,5$  e um ponto de sela para  $\alpha > 0,5$ .  $P_3$  é um ponto de sela para  $0 < \alpha < 1$  e um nó assintoticamente estável para  $\alpha > 1$ .  $P_4$  é um nó assintoticamente estável para  $0,5 < \alpha < 1$ .
18. (b)  $P_1(0,0), P_2(1,0), P_3(0; 0,75/\alpha), P_4[(4\alpha - 3)/(4\alpha - 2), 1/(4\alpha - 2)]$ .  $P_4$  está no primeiro quadrante para  $\alpha \geq 0,75$ .  
(c)  $\alpha = 0,75$ ;  $P_3$  coincide com  $P_4$ .
- (d)  $\mathbf{J} = \begin{pmatrix} 1 - 2x - y & -x \\ -0,5y & 0,75 - 2\alpha y - 0,5x \end{pmatrix}$
- (e)  $P_1$  é um nó instável.  $P_2$  é um ponto de sela.  $P_3$  é um nó assintoticamente estável para  $0 < \alpha < 0,75$  e um ponto de sela para  $\alpha > 0,75$ .  $P_4$  é um nó assintoticamente estável para  $\alpha > 0,75$ .
19. (b)  $P_1(0,0), P_2(1,0), P_3(0,\alpha), P_4(0,5; 0,5)$ . Além disso, para  $\alpha = 1$ , todo ponto na reta  $x + y = 1$  é um ponto crítico.  
(c)  $\alpha = 0$ ;  $P_3$  coincide com  $P_1$ . Também  $\alpha = 1$ .  
(d)  $\mathbf{J} = \begin{pmatrix} 1 - 2x - y & -x \\ -(2\alpha - 1)y & \alpha - 2y - (2\alpha - 1)x \end{pmatrix}$
- (e)  $P_1$  é um nó instável para  $\alpha > 0$ .  $P_2$  e  $P_3$  são pontos de sela para  $0 < \alpha < 1$  e nós assintoticamente estáveis para  $\alpha > 1$ .  $P_4$  é um ponto espiral assintoticamente estável para  $0 < \alpha < 0,5$ , um nó assintoticamente estável para  $0,5 < \alpha < 1$  e um ponto de sela para  $\alpha > 1$ .

### Seção 9.5

- (b, c)  $(0,0); u' = (\frac{3}{2})u, v' = (-\frac{1}{2})v; r = \frac{3}{2}, -\frac{1}{2}$ ; ponto de sela, instável  
 $(\frac{1}{2}, 3); u' = (-\frac{1}{4})v, v' = 3u; r = \pm\sqrt{3}i$ ; centro ou ponto espiral, indeterminado
- (b, c)  $(0,0); u' = u, v' = (-\frac{1}{4})v; r = 1, -\frac{1}{4}$ ; ponto de sela, instável  
 $(\frac{1}{2}, 2); u' = (-\frac{1}{4})v, v' = u; r = \pm(\frac{1}{2}i)$ ; centro ou ponto espiral, indeterminado
- (b, c)  $(0,0); u' = u, v' = (-\frac{1}{4})v; r = 1, -\frac{1}{4}$ ; ponto de sela, instável  
 $(2,0); u' = -u - v, v' = (\frac{3}{4})v; r = -1, \frac{3}{4}$ ; ponto de sela, instável  
 $(\frac{1}{2}, \frac{3}{2}); u' = (-\frac{1}{4})u - (\frac{1}{4})v, v' = (\frac{3}{4})u; r = (-1 \pm \sqrt{11}i)/8$ ; ponto espiral, assintoticamente estável
- (b, c)  $(0,0); u' = (\frac{9}{8})u, v' = -v; r = \frac{9}{8}, -1$ ; ponto de sela, instável  
 $(\frac{9}{8}, 0); u' = (-\frac{9}{8})u - (\frac{9}{16})v, v' = (\frac{1}{8})v; r = -\frac{9}{8}, \frac{1}{8}$ ; ponto de sela, instável  
 $(1, \frac{1}{4}); u' = -u - (\frac{1}{2})v, v' = (\frac{1}{4})u; r = (-1 \pm \sqrt{0,5})/2$ ; nó, assintoticamente estável
- (b, c)  $(0,0); u' = -u, v' = (-\frac{3}{2})v; r = -1, -\frac{3}{2}$ ; nó, assintoticamente estável  
 $(\frac{1}{2}, 0); u' = (\frac{3}{4})u - (\frac{3}{20})v, v' = -v; r = -1, \frac{3}{4}$ ; ponto de sela, instável  
 $(2,0); u' = -3u - (\frac{3}{5})v, v' = (\frac{1}{2})v; r = -3, \frac{1}{2}$ ; ponto de sela, instável  
 $(\frac{3}{2}, \frac{5}{3}); u' = (-\frac{3}{4})u - (\frac{9}{20})v, v' = (\frac{5}{3})u; r = (-3 \pm \sqrt{39}i)/8$ ; ponto espiral, assintoticamente estável
- (b, c)  $t = 0, T, 2T, \dots$ :  $H$  é um máximo,  $dP/dt$  é um máximo.  
 $t = T/4, 5T/4, \dots$ :  $dH/dt$  é um mínimo,  $P$  é um máximo.  
 $t = T/2, 3T/2, \dots$ :  $H$  é um mínimo,  $dP/dt$  é um mínimo.  
 $t = 3T/4, 7T/4, \dots$ :  $dH/dt$  é um máximo,  $P$  é um mínimo.
- (a)  $\sqrt{c}\alpha/\sqrt{a}\gamma$     (b)  $\sqrt{3}$   
(d) A razão das amplitudes da presa e do predador aumenta bem devagar quando o ponto inicial se afasta do ponto de equilíbrio.
- (a)  $4\pi/\sqrt{3} \cong 7,2552$   
(c) O período aumenta devagar quando o ponto inicial se afasta do ponto de equilíbrio.
- (a)  $T \cong 6,5$     (b)  $T \cong 3,7, T \cong 11,5$     (c)  $T \cong 3,8, T \cong 11,1$
- (a)  $P_1(0,0), P_2(1/\sigma, 0), P_3(3, 2 - 6\sigma)$ ;  $P_2$  se move para a esquerda e  $P_3$  se move para baixo; eles coincidem em  $(3, 0)$  quando  $\sigma = 1/3$ .  
(b)  $P_1$  é um ponto de sela.  $P_2$  é um ponto de sela para  $\sigma < 1/3$  e um nó assintoticamente estável para  $\sigma > 1/3$ .  $P_3$  é um ponto espiral assintoticamente estável para  $\sigma < \sigma_1 = (\sqrt{7/3} - 1)/2 \cong 0,2638$ , um nó assintoticamente estável para  $\sigma_1 < \sigma < 1/3$  e um ponto de sela para  $\sigma > 1/3$ .
- (a)  $P_1(0,0), P_2(a/\sigma, 0), P_3[c/\gamma, (a/\alpha) - (c\sigma/\alpha\gamma)]$ ;  $P_2$  se move para a esquerda e  $P_3$  se move para baixo; eles coincidem em  $(c/\gamma, 0)$  quando  $\sigma = a\gamma/c$ .  
(b)  $P_1$  é um ponto de sela.  $P_2$  é um ponto de sela para  $\sigma < a\gamma/c$  e um nó assintoticamente estável para  $\sigma > a\gamma/c$ .  $P_3$  é um ponto espiral assintoticamente estável para valores suficientemente pequenos de  $\sigma$  e torna-se um nó assintoticamente estável em um determinado valor  $\sigma_1 < a\gamma/c$ .  $P_3$  é um ponto de sela para  $\sigma > a\gamma/c$ .
- (a, b)  $P_1(0,0)$  é um ponto de sela;  $P_2(5,0)$  é um ponto de sela;  $P_3(2,2,4)$  é um ponto espiral assintoticamente estável.

14. (b) A mesma população de presas, menos predadores  
 (c) Mais presas, a mesma quantidade de predadores  
 (d) Mais presas, menos predadores
15. (b) A mesma população de presas, menos predadores  
 (c) Mais presas, menos predadores  
 (d) Mais presas, menos predadores ainda
16. (b) A mesma população de presas, menos predadores  
 (c) Mais presas, a mesma quantidade de predadores  
 (d) Mais presas, menos predadores

**Seção 9.7**

1.  $r = 1, \theta = t + t_0$ , ciclo limite estável.      2.  $r = 1, \theta = -t + t_0$ , ciclo limite semiestável
3.  $r = 1, \theta = t + t_0$ , ciclo limite estável;  $r = 3, \theta = t + t_0$ , solução periódica instável
4.  $r = 1, \theta = -t + t_0$ , solução periódica instável;  $r = 2, \theta = -t + t_0$ , ciclo limite estável
5.  $r = 2n - 1, \theta = t + t_0, n = 1, 2, 3, \dots$ , ciclo limite estável  
 $r = 2n, \theta = t + t_0, n = 1, 2, 3, \dots$ , solução periódica instável
6.  $r = 2, \theta = -t + t_0$ , ciclo limite semiestável  
 $r = 3, \theta = -t + t_0$ , solução periódica instável
8. (a) Sentido trigonométrico  
 (b)  $r = 1, \theta = t + t_0$ , ciclo limite estável;  $r = 2, \theta = t + t_0$ , ciclo limite semiestável;  $r = 3, \theta = t + t_0$ , solução periódica semiestável
9.  $r = \sqrt{2}, \theta = -t + t_0$ , solução periódica instável
14. (a)  $\mu = 0,2, T \cong 6,29$ ;  $\mu = 1, T \cong 6,66$ ;  $\mu = 5, T \cong 11,60$
15. (a)  $x' = y, y' = -x + \mu y - \mu y^3/3$   
 (b)  $0 < \mu < 2$ , ponto espiral instável;  $\mu \geq 2$ , nó instável  
 (c)  $A \cong 2,16, T \cong 6,65$   
 (d)  $\mu = 0,2, A \cong 1,99, T \cong 6,31$ ;  $\mu = 0,5, A \cong 2,03, T \cong 6,39$ ;  
 $\mu = 2, A \cong 2,60, T \cong 7,65$ ;  $\mu = 5, A \cong 4,36, T \cong 11,60$
16. (b)  $x' = \mu x + y, y' = -x + \mu y; \lambda = \mu \pm i$ ; a origem é um ponto espiral assintoticamente estável para  $\mu < 0$  e um ponto espiral instável para  $\mu > 0$   
 (c)  $r' = r(\mu - r^2), \theta' = -1$
17. (a) A origem é um nó assintoticamente estável para  $\mu < -2$ , um ponto espiral assintoticamente estável para  $-2 < \mu < 0$ , um ponto espiral instável para  $0 < \mu < 2$  e um nó instável para  $\mu > 2$ .
18. (a, b)  $(0,0)$  é um ponto de sela;  $(12,0)$  é um ponto de sela;  $(2,8)$  é um ponto espiral instável.
19. (a)  $(0,0), (5a,0), (2,4a-1,6)$   
 (b)  $r = -0,25 + 0,125a \pm 0,25\sqrt{220 - 400a + 25a^2}; a_0 = 2$
20. (b)  $\lambda = [-(5/4 - b) \pm \sqrt{(5/4 - b)^2 - 1}]/2$   
 (c)  $0 < b < 1/4$ : nó assintoticamente estável;  $1/4 < b < 5/4$ : ponto espiral assintoticamente estável;  $5/4 < b < 9/4$ : ponto espiral instável;  $9/4 < b$ : nó instável.  
 (d)  $b_0 = 5/4$
21. (b)  $k = 0, (1,1994, -0,62426)$ ;  $k = 0,5, (0,80485, -0,13106)$   
 (c)  $k_0 \cong 0,3465, (0,95450, -0,31813)$   
 (d)  $k = 0,4, T \cong 11,23$ ;  $k = 0,5, T \cong 10,37$ ;  $k = 0,6, T \cong 9,93$   
 (e)  $k_1 \cong 1,4035$

**Seção 9.8**

1. (b)  $\lambda = \lambda_1, \xi^{(1)} = (0,0,1)^T; \lambda = \lambda_2, \xi^{(2)} = (20,9 - \sqrt{81 + 40r}, 0)^T;$   
 $\lambda = \lambda_3, \xi^{(3)} = (20,9 + \sqrt{81 + 40r}, 0)^T$   
 (c)  $\lambda_1 \cong -2,6667, \xi^{(1)} = (0,0,1)^T; \lambda_2 \cong -22,8277, \xi^{(2)} \cong (20; -25,6554; 0)^T;$   
 $\lambda_3 \cong 11,8277, \xi^{(3)} \cong (20; 43,6554; 0)^T$
2. (c)  $\lambda_1 \cong -13,8546; \lambda_2, \lambda_3 \cong 0,0939556 \pm 10,1945i$
5. (a)  $dV/dt = -2\sigma[rx^2 + y^2 + b(z-r)^2 - br^2]$
11. (b)  $c = \sqrt{0,5} : P_1(\sqrt{2}/4, -\sqrt{2}, \sqrt{2}); \lambda = 0; -0,05178 \pm 1,5242i$   
 $c = 1 : P_1 = (0,8536; -3,4142; 3,4142); \lambda = 0,1612; -0,02882 \pm 2,0943i$   
 $P_2(0,1464; -0,5858; 0,5858); \lambda = -0,5303; -0,03665 \pm 1,1542i$
12. (a)  $P_1(1,1954; -4,7817; 4,7817); \lambda = 0,1893; -0,02191 \pm 2,4007i$   
 $P_2(0,1046; -0,4183; 0,4183); \lambda = -0,9614; 0,007964 \pm 1,0652i$   
 (d)  $T_1 \cong 5,9$

13. (a,b,c)  $c_1 \cong 1,243$   
 14. (a)  $P_1(2,9577; -11,8310; 11,8310); \lambda = 0,2273; -0,009796 \pm 3,5812i$   
 $P_2(0,04226; -0,1690; 0,1690); \lambda = -2,9053; 0,09877 \pm 0,9969i$   
 (c)  $T_2 \cong 11,8$   
 15. (a)  $P_1(3,7668; -15,0673; 15,0673); \lambda = 0,2324; -0,007814 \pm 4,0078i$   
 $P_2(0,03318; -0,1327; 0,1327); \lambda = -3,7335; 0,1083 \pm 0,9941i$   
 (b)  $T_4 \cong 23,6$

**CAPÍTULO 10****Seção 10.1**

1.  $y = -\sin x$       2.  $y = (\cot \sqrt{2}\pi \cos \sqrt{2}x + \sin \sqrt{2}x)/\sqrt{2}$   
 3.  $y = 0$  para todo  $L$ ;  $y = c_2 \sin x$  se  $\sin L = 0$   
 4.  $y = -\tan L \cos x + \sin x$  se  $\cos L \neq 0$ ; não solução se  $\cos L = 0$   
 5. Não tem solução      6.  $y = (-\pi \sin \sqrt{2}x + x \sin \sqrt{2}\pi)/2 \sin \sqrt{2}\pi$   
 7. Não tem solução      8.  $y = c_2 \sin 2x + \frac{1}{3} \sin x$   
 9.  $y = c_1 \cos 2x + \frac{1}{3} \cos x$       10.  $y = \frac{1}{2} \cos x$   
 11.  $y = -\frac{5}{2}x + \frac{3}{2}x^2$       12.  $y = -\frac{1}{9}x^{-1} + \frac{1}{9}(1 - e^3)x^{-1} \ln x + \frac{1}{9}x^2$   
 13. Não tem solução  
 14.  $\lambda_n = [(2n-1)/2]^2$ ,  $y_n(x) = \sin[(2n-1)x/2]$ ;  $n = 1, 2, 3, \dots$   
 15.  $\lambda_n = [(2n-1)/2]^2$ ,  $y_n(x) = \cos[(2n-1)x/2]$ ;  $n = 1, 2, 3, \dots$   
 16.  $\lambda_0 = 0$ ,  $y_0(x) = 1$ ;  $\lambda_n = n^2$ ,  $y_n(x) = \cos nx$ ;  $n = 1, 2, 3, \dots$   
 17.  $\lambda_n = [(2n-1)\pi/2L]^2$ ,  $y_n(x) = \cos[(2n-1)\pi x/2L]$ ;  $n = 1, 2, 3, \dots$   
 18.  $\lambda_0 = 0$ ,  $y_0(x) = 1$ ;  $\lambda_n = (n\pi/L)^2$ ,  $y_n(x) = \cos(n\pi x/L)$ ;  $n = 1, 2, 3, \dots$   
 19.  $\lambda_n = -[(2n-1)\pi/2L]^2$ ,  $y_n(x) = \sin[(2n-1)\pi x/2L]$ ;  $n = 1, 2, 3, \dots$   
 20.  $\lambda_n = 1 + (n\pi/\ln L)^2$ ,  $y_n(x) = x \sin(n\pi \ln x/\ln L)$ ;  $n = 1, 2, 3, \dots$   
 21. (a)  $w(r) = G(R^2 - r^2)/4\mu$       (c)  $Q$  é reduzido a 0,3164 de seu valor original  
 22. (a)  $y = k(x^4 - 2Lx^3 + L^3x)/24$   
 (b)  $y = k(x^4 - 2Lx^3 + L^2x^2)/24$   
 (c)  $y = k(x^4 - 4Lx^3 + 6L^2x^2)/24$

**Seção 10.2**

1.  $T = 2\pi/5$       2.  $T = 1$   
 3. Não é periódica      4.  $T = 2L$   
 5.  $T = 1$       6. Não é periódica  
 7.  $T = 2$       8.  $T = 4$   
 9.  $f(x) = 2L - x$  em  $L < x < 2L$ ;  $f(x) = -2L - x$  em  $-3L < x < -2L$   
 10.  $f(x) = x - 1$  em  $1 < x < 2$ ;  $f(x) = x - 8$  em  $8 < x < 9$   
 11.  $f(x) = -L - x$  em  $-L < x < 0$   
 13. (b)  $f(x) = \frac{2L}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi x}{L}$       14. (b)  $f(x) = \frac{1}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin[(2n-1)\pi x/L]}{2n-1}$   
 15. (b)  $f(x) = -\frac{\pi}{4} + \sum_{n=1}^{\infty} \left[ \frac{2 \cos(2n-1)x}{\pi(2n-1)^2} + \frac{(-1)^{n+1} \sin nx}{n} \right]$   
 16. (b)  $f(x) = \frac{1}{2} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos(2n-1)\pi x}{(2n-1)^2}$   
 17. (b)  $f(x) = \frac{3L}{4} + \sum_{n=1}^{\infty} \left[ \frac{2L \cos[(2n-1)\pi x/L]}{(2n-1)^2 \pi^2} + \frac{(-1)^{n+1} L \sin(n\pi x/L)}{n\pi} \right]$   
 18. (b)  $f(x) = \sum_{n=1}^{\infty} \left[ -\frac{2}{n\pi} \cos \frac{n\pi}{2} + \left( \frac{2}{n\pi} \right)^2 \sin \frac{n\pi}{2} \right] \sin \frac{n\pi x}{2}$   
 19. (b)  $f(x) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin[(2n-1)\pi x/2]}{2n-1}$       20. (b)  $f(x) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin n\pi x$   
 21. (b)  $f(x) = \frac{2}{3} + \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos \frac{n\pi x}{2}$   
 22. (b)  $f(x) = \frac{1}{2} + \frac{12}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)\pi x/2]}{(2n-1)^2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi x}{2}$

23. (b)  $f(x) = \frac{11}{12} + \frac{1}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n - 5}{n^2} \cos \frac{n\pi x}{2} + \sum_{n=1}^{\infty} \left[ \frac{4[1 - (-1)^n]}{n^3 \pi^3} - \frac{(-1)^n}{n\pi} \right] \sin \frac{n\pi x}{2}$
24. (b)  $f(x) = \frac{9}{8} + \sum_{n=1}^{\infty} \left[ \frac{162[(-1)^n - 1]}{n^4 \pi^4} - \frac{27(-1)^n}{n^2 \pi^2} \right] \cos \frac{n\pi x}{3} - \sum_{n=1}^{\infty} \frac{108(-1)^n + 54}{n^3 \pi^3} \sin \frac{n\pi x}{3}$
25. (b)  $m = 81$
26. (b)  $m = 27$
28.  $\int_0^x f(t) dt$  pode não ser periódica; por exemplo, seja  $f(t) = 1 + \cos t$ .

**Seção 10.3**

1. (a)  $f(x) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)\pi x}{2n-1}$
2. (a)  $f(x) = \frac{\pi}{4} - \sum_{n=1}^{\infty} \left[ \frac{2}{(2n-1)^2 \pi} \cos(2n-1)x + \frac{(-1)^n}{n} \sin nx \right]$
3. (a)  $f(x) = \frac{L}{2} + \frac{4L}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)\pi x/L]}{(2n-1)^2}$
4. (a)  $f(x) = \frac{2}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos n\pi x$
5. (a)  $f(x) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos(2n-1)x$
6. (a)  $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi x + b_n \sin n\pi x);$   
 $a_0 = \frac{1}{3}, \quad a_n = \frac{2(-1)^n}{n^2 \pi^2}, \quad b_n = \begin{cases} -1/n\pi, & n \text{ par} \\ 1/n\pi - 4/n^3 \pi^3, & n \text{ ímpar} \end{cases}$
7. (a)  $f(x) = -\frac{\pi}{4} + \sum_{n=1}^{\infty} \left[ \frac{1 - \cos n\pi}{\pi n^2} \cos nx - \frac{(-1)^n}{n} \sin nx \right]$   
 (b)  $n = 10; \max|e| = 1,6025 \text{ em } x = \pm\pi$   
 $n = 20; \max|e| = 1,5867 \text{ em } x = \pm\pi$   
 $n = 40; \max|e| = 1,5788 \text{ em } x = \pm\pi$   
 (c) Não é possível
8. (a)  $f(x) = \frac{1}{2} + \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{1 - \cos n\pi}{n^2} \cos n\pi x$   
 (b)  $n = 10; \max|e| = 0,02020 \text{ em } x = 0, \pm 1$   
 $n = 20; \max|e| = 0,01012 \text{ em } x = 0, \pm 1$   
 $n = 40; \max|e| = 0,005065 \text{ em } x = 0, \pm 1$   
 (c)  $n = 21$
9. (a)  $f(x) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin n\pi x$   
 (b)  $n = 10, 20, 40; \max|e| = 1 \text{ em } x = \pm 1$   
 (c) Não é possível
10. (a)  $f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \left[ \frac{6(1 - \cos n\pi)}{n^2 \pi^2} \cos \frac{n\pi x}{2} + \frac{2 \cos n\pi}{n\pi} \sin \frac{n\pi x}{2} \right]$   
 (b)  $n = 10; \sup|e| = 1,0606 \text{ quando } x \rightarrow 2$   
 $n = 20; \sup|e| = 1,0304 \text{ quando } x \rightarrow 2$   
 $n = 40; \sup|e| = 1,0152 \text{ quando } x \rightarrow 2$   
 (c) Não é possível
11. (a)  $f(x) = \frac{1}{6} + \sum_{n=1}^{\infty} \left[ \frac{2 \cos n\pi}{n^2 \pi^2} \cos n\pi x - \frac{2 - 2 \cos n\pi + n^2 \pi^2 \cos n\pi}{n^3 \pi^3} \sin n\pi x \right]$   
 (b)  $n = 10; \sup|e| = 0,5193 \text{ quando } x \rightarrow 1$   
 $n = 20; \sup|e| = 0,5099 \text{ quando } x \rightarrow 1$   
 $n = 40; \sup|e| = 0,5050 \text{ quando } x \rightarrow 1$   
 (c) Não é possível

12. (a)  $f(x) = -\frac{12}{\pi^3} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \sin n\pi x$
- (b)  $n = 10; \max|e| = 0,001345 \text{ em } x = \pm 0,9735$   
 $n = 20; \max|e| = 0,0003534 \text{ em } x = \pm 0,9864$   
 $n = 40; \max|e| = 0,00009058 \text{ em } x = \pm 0,9931$
- (c)  $n = 4$
13.  $y = (\omega \sin nt - n \sin \omega t)/\omega(\omega^2 - n^2), \quad \omega^2 \neq n^2$   
 $y = (\sin nt - nt \cos nt)/2n^2, \quad \omega^2 = n^2$
14.  $y = \sum_{n=1}^{\infty} b_n (\omega \sin nt - n \sin \omega t)/\omega(\omega^2 - n^2), \quad \omega \neq 1, 2, 3, \dots$   
 $y = \sum_{\substack{n=1 \\ n \neq m}}^{\infty} b_n (m \sin nt - n \sin mt)/m(m^2 - n^2) + b_m (\sin mt - mt \cos mt)/2m^2, \quad \omega = n$
15.  $y = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{\omega^2 - (2n-1)^2} \left[ \frac{1}{2n-1} \sin(2n-1)t - \frac{1}{\omega} \sin \omega t \right]$
16.  $y = \cos \omega t + \frac{1}{2\omega^2} (1 - \cos \omega t) + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos(2n-1)\pi t - \cos \omega t}{(2n-1)^2 [\omega^2 - (2n-1)^2 \pi^2]}$

#### Seção 10.4

1. Ímpar  
 2. Nenhuma das duas  
 3. Ímpar  
 4. Par  
 5. Par  
 6. Nenhuma das duas
14.  $f(x) = \frac{1}{4} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1 - \cos(n\pi/2)}{n^2} \cos \frac{n\pi x}{2}$   
 $f(x) = \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(n\pi/2) - \sin(n\pi/2)}{n^2} \sin \frac{n\pi x}{2}$
15. (a)  $f(x) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos \frac{(2n-1)\pi x}{2}$
16. (a)  $f(x) = \sum_{n=1}^{\infty} \frac{2}{n\pi} \left( -\cos n\pi + \frac{2}{n\pi} \sin \frac{n\pi}{2} \right) \sin \frac{n\pi x}{2}$
17. (a)  $f(x) = 1$   
 18. (a)  $f(x) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{2n-1}$
19. (a)  $f(x) = \sum_{n=1}^{\infty} \frac{2}{n\pi} \left( \cos \frac{n\pi}{3} + \cos \frac{2n\pi}{3} - 2 \cos n\pi \right) \sin \frac{nx}{3}$
20. (a)  $f(x) = \frac{1}{2} - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{\sin 2n\pi x}{n}$   
 21. (a)  $f(x) = \frac{L}{2} + \frac{4L}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)\pi x/L]}{(2n-1)^2}$
22. (a)  $f(x) = \frac{2L}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\pi x/L)}{n}$
23. (a)  $f(x) = \frac{\pi}{4} + \frac{1}{\pi} \sum_{n=1}^{\infty} \left[ \frac{2\pi}{n} \sin \frac{n\pi}{2} + \frac{4}{n^2} \left( \cos \frac{n\pi}{2} - 1 \right) \right] \cos \frac{nx}{2}$
24. (a)  $f(x) = 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin nx$
25. (a)  $f(x) = \sum_{n=1}^{\infty} \left[ \frac{4n^2\pi^2(1 + \cos n\pi)}{n^3\pi^3} + \frac{16(1 - \cos n\pi)}{n^3\pi^3} \right] \sin \frac{n\pi x}{2}$
26. (a)  $f(x) = \frac{4}{3} + \frac{16}{\pi^2} \sum_{n=1}^{\infty} \frac{1 + 3 \cos n\pi}{n^2} \cos \frac{n\pi x}{4}$
27. (b)  $g(x) = \frac{3}{2} + \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1 - \cos n\pi}{n^2} \cos \frac{n\pi x}{3}$   
 $h(x) = \frac{6}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi x}{3}$

28. (b)  $g(x) = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{4 \cos(n\pi/2) + 2n\pi \sin(n\pi/2) - 4}{n^2\pi^2} \cos \frac{n\pi x}{2}$

$$h(x) = \sum_{n=1}^{\infty} \frac{4 \sin(n\pi/2) - 2n\pi \cos(n\pi/2)}{n^2\pi^2} \sin \frac{n\pi x}{2}$$

29. (b)  $g(x) = -\frac{5}{12} + \sum_{n=1}^{\infty} \frac{12 \cos n\pi + 4}{n^2\pi^2} \cos \frac{n\pi x}{2}$

$$h(x) = -\frac{1}{2} \sum_{n=1}^{\infty} \frac{n^2\pi^2(3 + 5 \cos n\pi) + 32(1 - \cos n\pi)}{n^3\pi^3} \sin \frac{n\pi x}{2}$$

30. (b)  $g(x) = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{6n^2\pi^2(2 \cos n\pi - 5) + 324(1 - \cos n\pi)}{n^4\pi^4} \cos \frac{n\pi x}{3}$

$$h(x) = \sum_{n=1}^{\infty} \left[ \frac{4 \cos n\pi + 2}{n\pi} + \frac{144 \cos n\pi + 180}{n^3\pi^3} \right] \sin \frac{n\pi x}{3}$$

40. (a) Estenda  $f(x)$  antissimetricamente a  $(L, 2L)$ , ou seja, de modo que  $f(2L - x) = -f(x)$  para  $0 \leq x < L$ . Depois, estenda esta função como uma função par a  $(-2L, 0)$ .

### Seção 10.5

1.  $xX'' - \lambda X = 0, \quad T' + \lambda tT = 0$
2.  $X'' - \lambda xX = 0, \quad T' + \lambda tT = 0$
3.  $X'' - \lambda(X' + X) = 0, \quad T' + \lambda T = 0$
4.  $[p(x)X']' + \lambda r(x)X = 0, \quad T'' + \lambda T = 0$
5. Não é separável
6.  $X'' + (x + \lambda)X = 0, \quad Y'' - \lambda Y = 0$
7.  $u(x, t) = e^{-400\pi^2 t} \sin 2\pi x - e^{-2500\pi^2 t} \sin 5\pi x$
8.  $u(x, t) = 2e^{-\pi^2 t/16} \sin(\pi x/2) - e^{-\pi^2 t/4} \sin \pi x + 4e^{-\pi^2 t} \sin 2\pi x$
9.  $u(x, t) = \frac{100}{\pi} \sum_{n=1}^{\infty} \frac{1 - \cos n\pi}{n} e^{-n^2\pi^2 t/1600} \sin \frac{n\pi x}{40}$
10.  $u(x, t) = \frac{160}{\pi^2} \sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n^2} e^{-n^2\pi^2 t/1600} \sin \frac{n\pi x}{40}$
11.  $u(x, t) = \frac{100}{\pi} \sum_{n=1}^{\infty} \frac{\cos(n\pi/4) - \cos(3n\pi/4)}{n} e^{-n^2\pi^2 t/1600} \sin \frac{n\pi x}{40}$
12.  $u(x, t) = \frac{80}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-n^2\pi^2 t/1600} \sin \frac{n\pi x}{40}$
13.  $t = 5, n = 16; \quad t = 20, n = 8; \quad t = 80, n = 4$
14. (d)  $t = 673,35$
15. (d)  $t = 451,60$
16. (d)  $t = 617,17$
17. (b)  $t = 5, x = 33,20; \quad t = 10, x = 31,13; \quad t = 20, x = 28,62; \quad t = 40, x = 25,73;$   
 $t = 100, x = 21,95; \quad t = 200, x = 20,31$
- (e)  $t = 524,81$
18.  $u(x, t) = \frac{200}{\pi} \sum_{n=1}^{\infty} \frac{1 - \cos n\pi}{n} e^{-n^2\pi^2 a^2 t/400} \sin \frac{n\pi x}{20}$ 
  - (a)  $35,91^\circ\text{C}$
  - (b)  $67,23^\circ\text{C}$
  - (c)  $99,96^\circ\text{C}$
19. (a)  $76,73 \text{ s}$
20. (b)  $152,56 \text{ s}$
21. (a)  $aw_{xx} - bw_t + (c - b\delta)w = 0$
22.  $X'' + \mu^2 X = 0, \quad Y'' + (\lambda^2 - \mu^2)Y = 0, \quad T' + \alpha^2 \lambda^2 T = 0$
23.  $r^2 R'' + rR' + (\lambda^2 r^2 - \mu^2)R = 0, \quad \Theta'' + \mu^2 \Theta = 0, \quad T' + \alpha^2 \lambda^2 T = 0$

### Seção 10.6

1.  $u = 10 + \frac{3}{5}x$
2.  $u = 30 - \frac{5}{4}x$
3.  $u = 0$
4.  $u = T$
5.  $u = 0$
6.  $u = T$
7.  $u = T(1 + x)/(1 + L)$
8.  $u = T(1 + L - x)/(1 + L)$
9. (a)  $u(x, t) = 3x + \sum_{n=1}^{\infty} \frac{70 \cos n\pi + 50}{n\pi} e^{-0.86n^2\pi^2 t/400} \sin \frac{n\pi x}{20}$
- (d)  $160,29 \text{ s}$

10. (a)  $f(x) = 2x$ ,  $0 \leq x \leq 50$ ;  $f(x) = 200 - 2x$ ,  $50 < x \leq 100$

(b)  $u(x, t) = 20 - \frac{x}{5} + \sum_{n=1}^{\infty} c_n e^{-1.14n^2\pi^2t/(100)^2} \sin \frac{n\pi x}{100}$ ,

$$c_n = \frac{800}{n^2\pi^2} \sin \frac{n\pi}{2} - \frac{40}{n\pi}$$

(d)  $u(50, t) \rightarrow 10$  quando  $t \rightarrow \infty$ ; 3754 s

11. (a)  $u(x, t) = 30 - x + \sum_{n=1}^{\infty} c_n e^{-n^2\pi^2t/900} \sin \frac{n\pi x}{30}$ ,

$$c_n = \frac{60}{n^3\pi^3} [2(1 - \cos n\pi) - n^2\pi^2(1 + \cos n\pi)]$$

12. (a)  $u(x, t) = \frac{2}{\pi} + \sum_{n=1}^{\infty} c_n e^{-n^2\pi^2a^2t/L^2} \cos \frac{n\pi x}{L}$ ,

$$c_n = \begin{cases} 0, & n \text{ ímpar;} \\ -4/(n^2 - 1)\pi, & n \text{ par} \end{cases}$$

(b)  $\lim_{t \rightarrow \infty} u(x, t) = 2/\pi$

13. (a)  $u(x, t) = \frac{200}{9} + \sum_{n=1}^{\infty} c_n e^{-n^2\pi^2t/6400} \cos \frac{n\pi x}{40}$ ,

$$c_n = -\frac{160}{3n^2\pi^2} (3 + \cos n\pi)$$

(c) 200/9

(d) 1543 s

14. (a)  $u(x, t) = \frac{25}{6} + \sum_{n=1}^{\infty} c_n e^{-n^2\pi^2t/900} \cos \frac{n\pi x}{30}$ ,

$$c_n = \frac{50}{n\pi} \left( \sin \frac{n\pi}{3} - \sin \frac{n\pi}{6} \right)$$

15. (b)  $u(x, t) = \sum_{n=1}^{\infty} c_n e^{-(2n-1)^2\pi^2a^2t/4L^2} \sin \frac{(2n-1)\pi x}{2L}$ ,

$$c_n = \frac{2}{L} \int_0^L f(x) \sin \frac{(2n-1)\pi x}{2L} dx$$

16. (a)  $u(x, t) = \sum_{n=1}^{\infty} c_n e^{-(2n-1)^2\pi^2t/3600} \sin \frac{(2n-1)\pi x}{60}$ ,

$$c_n = \frac{120}{(2n-1)^2\pi^2} [2 \cos n\pi + (2n-1)\pi]$$

(c)  $x_m$  aumenta a partir de  $x = 0$  e chega a  $x = 30$  quando  $t = 104.4$ .

17. (a)  $u(x, t) = 40 + \sum_{n=1}^{\infty} c_n e^{-(2n-1)^2\pi^2t/3600} \sin \frac{(2n-1)\pi x}{60}$ ,

$$c_n = \frac{40}{(2n-1)^2\pi^2} [6 \cos n\pi - (2n-1)\pi]$$

19.  $u(x) = \begin{cases} T \frac{x}{a} \left[ \frac{\xi}{\xi + (L/a) - 1} \right], & 0 \leq x \leq a, \\ T \left[ 1 - \frac{L-x}{a} \frac{1}{\xi + (L/a) - 1} \right], & a \leq x \leq L, \end{cases}$  onde  $\xi = \kappa_2 A_2 / \kappa_1 A_1$

20. (e)  $u_n(x, t) = e^{-\mu_n^2 a^2 t} \sin \mu_n x$

21.  $\alpha^2 v'' + s(x) = 0$ ;  $v(0) = T_1$ ,  $v(L) = T_2$

$$w_t = \alpha^2 w_{xx}; \quad w(0, t) = 0, \quad w(L, t) = 0, \quad w(x, 0) = f(x) - v(x)$$

22. (a)  $v(x) = T_1 + (T_2 - T_1)(x/L) + kLx/2 - kx^2/2$

(b)  $w(x, t) = \sum_{n=1}^{\infty} c_n e^{-n^2\pi^2t/400} \sin \frac{n\pi x}{20}$ ,  $c_n = \frac{160(\cos n\pi - 1)}{n^3\pi^3}$

23. (a)  $v(x) = T_1 + (T_2 - T_1)x/L + kLx/6 - kx^3/6L$

(b)  $w(x, t) = \sum_{n=1}^{\infty} c_n e^{-n^2\pi^2t/400} \sin \frac{n\pi x}{20}$ ,

$$c_n = \frac{20}{3} \left[ \frac{3m^3\pi^3(3 \cos m\pi - 1) + 60 \cos m\pi}{m^4\pi^4} \right]$$

**Seção 10.7**

1. (a)  $u(x, t) = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{n\pi}{2} \sin \frac{n\pi x}{L} \cos \frac{n\pi at}{L}$

2. (a)  $u(x, t) = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left( \sin \frac{n\pi}{4} + \sin \frac{3n\pi}{4} \right) \sin \frac{n\pi x}{L} \cos \frac{n\pi at}{L}$

3. (a)  $u(x, t) = \frac{32}{\pi^3} \sum_{n=1}^{\infty} \frac{2 + \cos n\pi}{n^3} \sin \frac{n\pi x}{L} \cos \frac{n\pi at}{L}$

4. (a)  $u(x, t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\pi/2) \sin(n\pi/L)}{n} \sin \frac{n\pi x}{L} \cos \frac{n\pi at}{L}$

5. (a)  $u(x, t) = \frac{8L}{a\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^3} \sin \frac{n\pi}{2} \sin \frac{n\pi x}{L} \sin \frac{n\pi at}{L}$

6. (a)  $u(x, t) = \frac{8L}{a\pi^3} \sum_{n=1}^{\infty} \frac{\sin(n\pi/4) + \sin(3n\pi/4)}{n^3} \sin \frac{n\pi x}{L} \sin \frac{n\pi at}{L}$

7. (a)  $u(x, t) = \frac{32L}{a\pi^4} \sum_{n=1}^{\infty} \frac{\cos n\pi + 2}{n^4} \sin \frac{n\pi x}{L} \sin \frac{n\pi at}{L}$

8. (a)  $u(x, t) = \frac{4L}{a\pi^2} \sum_{n=1}^{\infty} \frac{\sin(n\pi/2) \sin(n\pi/L)}{n^2} \sin \frac{n\pi x}{L} \sin \frac{n\pi at}{L}$

9.  $u(x, t) = \sum_{n=1}^{\infty} c_n \sin \frac{(2n-1)\pi x}{2L} \cos \frac{(2n-1)\pi at}{2L},$

$$c_n = \frac{2}{L} \int_0^L f(x) \sin \frac{(2n-1)\pi x}{2L} dx$$

10. (a)  $u(x, t) = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin \frac{(2n-1)\pi}{4} \sin \frac{(2n-1)\pi x}{2L} \sin \frac{(2n-1)\pi x}{2L} \cos \frac{(2n-1)\pi at}{2L}$

11. (a)  $u(x, t) = \frac{512}{\pi^4} \sum_{n=1}^{\infty} \frac{(2n-1)\pi + 3 \cos n\pi}{(2n-1)^4} \sin \frac{(2n-1)\pi x}{2L} \cos \frac{(2n-1)\pi at}{2L}$

14. (b)  $\phi(x+at)$  representa uma onda movendo-se no sentido negativo de  $x$  com velocidade  $a > 0$ .

15. (a) 248 ft/s (b)  $49,6\pi n$  rad/s (c) As frequências aumentam; os modos permanecem inalterados.

21.  $r^2 R'' + rR' + (\lambda^2 r^2 - \mu^2)R = 0, \quad \Theta'' + \mu^2 \Theta = 0, \quad T'' + \lambda^2 a^2 T = 0$

23. (b)  $a_n = a\sqrt{1 + (\gamma^2 L^2/n^2\pi^2)}$  (c)  $\gamma = 0$

24. (a)  $c_n = \frac{20}{n^2\pi^2} \left( 2 \sin \frac{n\pi}{2} - \sin \frac{2n\pi}{5} - \sin \frac{3n\pi}{5} \right)$

**Seção 10.8**

1. (a)  $u(x, y) = \sum_{n=1}^{\infty} c_n \sin \frac{n\pi x}{a} \operatorname{senh} \frac{n\pi y}{a}, \quad c_n = \frac{2/a}{\operatorname{senh}(n\pi b/a)} \int_0^a g(x) \sin \frac{n\pi x}{a} dx$

(b)  $u(x, y) = \frac{4a}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \frac{\sin(n\pi/2)}{\operatorname{senh}(n\pi b/a)} \sin \frac{n\pi x}{a} \operatorname{senh} \frac{n\pi y}{a}$

2.  $u(x, y) = \sum_{n=1}^{\infty} c_n \sin \frac{n\pi x}{a} \operatorname{senh} \frac{n\pi(b-y)}{a}, \quad c_n = \frac{2/a}{\operatorname{senh}(n\pi b/a)} \int_0^a h(x) \sin \frac{n\pi x}{a} dx$

3. (a)  $u(x, y) = \sum_{n=1}^{\infty} c_n^{(1)} \operatorname{senh} \frac{n\pi x}{b} \sin \frac{n\pi y}{b} + \sum_{n=1}^{\infty} c_n^{(2)} \sin \frac{n\pi x}{a} \operatorname{senh} \frac{n\pi(b-y)}{a},$

$$c_n^{(1)} = \frac{2/b}{\operatorname{senh}(n\pi a/b)} \int_0^b f(y) \sin \frac{n\pi y}{b} dy, \quad c_n^{(2)} = \frac{2/a}{\operatorname{senh}(n\pi b/a)} \int_0^a h(x) \sin \frac{n\pi x}{a} dx$$

(b)  $c_n^{(1)} = \frac{2}{n\pi \operatorname{senh}(n\pi a/b)}, \quad c_n^{(2)} = -\frac{2}{n^3\pi^3} \frac{(n^2\pi^2 - 2) \cos n\pi + 2}{\operatorname{senh}(n\pi b/a)}$

5.  $u(r, \theta) = \frac{c_0}{2} + \sum_{n=1}^{\infty} r^{-n} (c_n \cos n\theta + k_n \sin n\theta);$

$$c_n = \frac{a^n}{\pi} \int_0^{2\pi} f(\theta) \cos n\theta d\theta, \quad k_n = \frac{a^n}{\pi} \int_0^{2\pi} f(\theta) \sin n\theta d\theta$$

6. (a)  $u(r, \theta) = \sum_{n=1}^{\infty} c_n r^n \operatorname{sen} n\theta$ ,  $c_n = \frac{2}{\pi a^n} \int_0^\pi f(\theta) \operatorname{sen} n\theta d\theta$   
(b)  $c_n = \frac{4}{\pi a^n} \frac{1 - \cos n\pi}{n^3}$
7.  $u(r, \theta) = \sum_{n=1}^{\infty} c_n r^{n\pi/a} \operatorname{sen} \frac{n\pi\theta}{\alpha}$ ,  $c_n = (2/\alpha) a^{-n\pi/a} \int_0^\alpha f(\theta) \operatorname{sen} \frac{n\pi\theta}{\alpha} d\theta$
8. (a)  $u(x, y) = \sum_{n=1}^{\infty} c_n e^{-n\pi y/a} \operatorname{sen} \frac{n\pi x}{a}$ ,  $c_n = \frac{2}{a} \int_0^a f(x) \operatorname{sen} \frac{n\pi x}{a} dx$   
(b)  $c_n = \frac{4a^2}{n^3 \pi^3} (1 - \cos n\pi)$  (c)  $y_0 \cong 6,6315$
10. (b)  $u(x, y) = c_0 + \sum_{n=1}^{\infty} c_n \cosh \frac{n\pi x}{b} \cos \frac{n\pi y}{b}$ ,  $c_n = \frac{2/n\pi}{\operatorname{senh}(n\pi a/b)} \int_0^b f(y) \cos \frac{n\pi y}{b} dy$
11.  $u(r, \theta) = c_0 + \sum_{n=1}^{\infty} r^n (c_n \cos n\theta + k_n \operatorname{sen} n\theta)$ ,  
 $c_n = \frac{1}{n\pi a^{n-1}} \int_0^{2\pi} g(\theta) \cos n\theta d\theta$ ,  $k_n = \frac{1}{n\pi a^{n-1}} \int_0^{2\pi} g(\theta) \operatorname{sen} n\theta d\theta$ ;  
a condição necessária é  $\int_0^{2\pi} g(\theta) d\theta = 0$ .
12. (a)  $u(x, y) = \sum_{n=1}^{\infty} c_n \operatorname{sen} \frac{n\pi x}{a} \cosh \frac{n\pi y}{a}$ ,  $c_n = \frac{2/a}{\cosh(n\pi b/a)} \int_0^a g(x) \operatorname{sen} \frac{n\pi x}{a} dx$   
(b)  $c_n = \frac{4a \operatorname{sen}(n\pi/2)}{n^2 \pi^2 \cosh(n\pi b/a)}$
13. (a)  $u(x, y) = \sum_{n=1}^{\infty} c_n \operatorname{senh} \frac{(2n-1)\pi x}{2b} \operatorname{sen} \frac{(2n-1)\pi y}{2b}$ ,  
 $c_n = \frac{2/b}{\operatorname{senh}[(2n-1)\pi a/2b]} \int_0^b f(y) \operatorname{sen} \frac{(2n-1)\pi y}{2b} dy$   
(b)  $c_n = \frac{32b^2}{(2n-1)^3 \pi^3 \operatorname{senh}[(2n-1)\pi a/2b]}$
14. (a)  $u(x, y) = \frac{c_0 y}{2} + \sum_{n=1}^{\infty} c_n \cos \frac{n\pi x}{a} \operatorname{senh} \frac{n\pi y}{a}$ ,  
 $c_0 = \frac{2}{ab} \int_0^a g(x) dx$ ,  $c_n = \frac{2/a}{\operatorname{senh}(n\pi b/a)} \int_0^a g(x) \cos \frac{n\pi x}{a} dx$   
(b)  $c_0 = \frac{2}{b} \left(1 + \frac{a^4}{30}\right)$ ,  $c_n = -\frac{24a^4(1 + \cos n\pi)}{n^4 \pi^4 \operatorname{senh}(n\pi b/a)}$
16. (a)  $u(x, z) = b + \frac{\alpha a}{2} - \frac{4\alpha a}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)\pi x/a] \cosh[(2n-1)\pi z/a]}{(2n-1)^2 \cosh[(2n-1)\pi b/a]}$

**CAPÍTULO 11****Seção 11.1**

1. Homogênea
2. Não homogênea
3. Não homogênea
4. Homogênea
5. Não homogênea
6. Homogênea
7. (a)  $\phi_n(x) = \operatorname{sen} \sqrt{\lambda_n} x$ , onde  $\sqrt{\lambda_n}$  satisfaz  $\sqrt{\lambda} = -\tan \sqrt{\lambda} \pi$ ; (b) Não  
(c)  $\lambda_1 \cong 0,6204$ ,  $\lambda_2 \cong 2,7943$   
(d)  $\lambda_n \cong (2n-1)^2/4$  para  $n$  grande
8. (a)  $\phi_n(x) = \cos \sqrt{\lambda_n} x$ , onde  $\sqrt{\lambda_n}$  satisfaz  $\sqrt{\lambda} = \cot \sqrt{\lambda}$ ; (b) Não  
(c)  $\lambda_1 \cong 0,7402$ ,  $\lambda_2 \cong 11,7349$   
(d)  $\lambda_n \cong (n-1)^2 \pi^2$  para  $n$  grande
9. (a)  $\phi_n(x) = \operatorname{sen} \sqrt{\lambda_n} x + \sqrt{\lambda_n} \cos \sqrt{\lambda_n} x$ , onde  $\sqrt{\lambda_n}$  satisfaz  $(\lambda - 1) \operatorname{sen} \sqrt{\lambda} - 2\sqrt{\lambda} \cos \sqrt{\lambda} = 0$ ; (b) Não  
(c)  $\lambda_1 \cong 1,7071$ ,  $\lambda_2 \cong 13,4924$   
(d)  $\lambda_n \cong (n-1)^2 \pi^2$  para  $n$  grande
10. (a) Para  $n = 1, 2, 3, \dots$ ,  $\phi_n(x) = \operatorname{sen} \mu_n x - \mu_n \cos \mu_n x$  e  $\lambda_n = -\mu_n^2$ , onde  $\mu_n$  satisfaz  $\mu = \tan \mu$ .  
(b) Sim;  $\lambda_0 = 0$ ,  $\phi_0(x) = 1 - x$

- (c)  $\lambda_1 \cong -20,1907$ ,  $\lambda_2 \cong -59,6795$   
 (d)  $\lambda_n \cong -(2n+1)^2\pi^2/4$  para  $n$  grande
12.  $\mu(x) = e^{-x^2}$       13.  $\mu(x) = 1/x$   
 14.  $\mu(x) = e^{-x}$       15.  $\mu(x) = (1-x^2)^{-1/2}$   
 16.  $X'' + \lambda X = 0$ ,  $T'' + cT' + (k + \lambda a^2)T = 0$   
 17. (a)  $s(x) = e^x$       (b)  $\lambda_n = n^2\pi^2$ ,  $\phi_n(x) = e^x \operatorname{sen} n\pi x$ ;  $n = 1, 2, 3, \dots$   
 18. Os autovalores positivos são  $\lambda = \lambda_n$ , onde  $\sqrt{\lambda_n}$  satisfaz  $\sqrt{\lambda} = (\frac{2}{3})\tan(3\sqrt{\lambda}L)$ ; as autofunções associadas são  $\phi_n(x) = e^{-2x}\operatorname{sen}(3\sqrt{\lambda_n}x)$ . Se  $L = \frac{1}{2}$ ,  $\lambda_0 = 0$  é um autovalor com autofunção associada  $\phi_0(x) = xe^{-2x}$ ; se  $L \neq \frac{1}{2}$ ,  $\lambda = 0$  não é autovalor. Se  $L \leq \frac{1}{2}$ , não existem autovalores negativos; se  $L > \frac{1}{2}$ , existe um autovalor negativo  $\lambda = -\mu^2$ , onde  $\mu$  é uma raiz de  $\mu = (\frac{2}{3})\tanh(3\mu L)$ ; a autofunção associada é  $\phi_{-1}(x) = e^{-2x}\operatorname{senh}(3\mu x)$ .  
 19. Não tem autovalores reais.  
 20. O único autovalor é  $\lambda = 0$ ; a autofunção associada é  $\phi(x) = x - 1$ .  
 21. (a)  $2\operatorname{sen}\sqrt{\lambda} - \sqrt{\lambda}\cos\sqrt{\lambda} = 0$   
 (c)  $\lambda_1 \cong 18,2738$ ,  $\lambda_2 \cong 57,7075$   
 (d)  $2\operatorname{senh}\sqrt{\mu} - \sqrt{\mu}\cosh\sqrt{\mu} = 0$ ,  $\mu = -\lambda$   
 (e)  $\lambda_{-1} \cong -3,6673$   
 24. (a)  $\lambda_n = \mu_n^4$ , onde  $\mu_n$  é uma raiz de  $\operatorname{sen}\mu L \operatorname{senh}\mu L = 0$ , logo  $\lambda_n = (n\pi/L)^4$ ;  
 $\lambda_1 \cong 97,409/L^4$ ,  $\lambda_2 \cong 1558,5/L^4$ ,  $\phi_n(x) = \operatorname{sen}(n\pi x/L)$   
 (b)  $\lambda_n = \mu_n^4$ , onde  $\mu_n$  é uma raiz de  $\operatorname{sen}\mu L \cosh\mu L - \cos\mu L \operatorname{senh}\mu L = 0$ ;  
 $\lambda_1 \cong 237,72/L^4$ ,  $\lambda_2 \cong 2496,5/L^4$ ,  $\phi_n = \frac{\operatorname{sen}\mu_n x \operatorname{senh}\mu_n L - \operatorname{sen}\mu_n L \operatorname{senh}\mu_n x}{\operatorname{senh}\mu_n L}$   
 (c)  $\lambda_n = \mu_n^4$ , onde  $\mu_n$  é uma raiz de  $1 + \cosh\mu L \cos\mu L = 0$ :  $\lambda_1 \cong 12,362/L^4$ ,  
 $\lambda_2 \cong 485,52/L^4$   
 $\phi_n(x) = \frac{[(\operatorname{sen}\mu_n x - \operatorname{senh}\mu_n x)(\cos\mu_n L + \cosh\mu_n L) + (\operatorname{sen}\mu_n L + \operatorname{senh}\mu_n L)(\cosh\mu_n x - \cos\mu_n x)]}{\cos\mu_n L + \cosh\mu_n L}$   
 25. (c)  $\phi_n(x) = \operatorname{sen}\sqrt{\lambda_n}x$ , onde  $\lambda_n$  satisfaz  $\cos\sqrt{\lambda_n}L - \gamma\sqrt{\lambda_n}L\operatorname{sen}\sqrt{\lambda_n}L = 0$   
 (d)  $\lambda_1 \cong 1,1597/L^2$ ,  $\lambda_2 \cong 13,276/L^2$

### Seção 11.2

1.  $\phi_n(x) = \sqrt{2}\operatorname{sen}(n - \frac{1}{2})\pi x$ ;  $n = 1, 2, \dots$       2.  $\phi_n(x) = \sqrt{2}\cos(n - \frac{1}{2})\pi x$ ;  $n = 1, 2, \dots$   
 3.  $\phi_0(x) = 1$ ,  $\phi_n(x) = \sqrt{2}\cos n\pi x$ ;  $n = 1, 2, \dots$   
 4.  $\phi_n(x) = \frac{\sqrt{2}\cos\sqrt{\lambda_n}x}{(1 + \operatorname{sen}^2\sqrt{\lambda_n})^{1/2}}$ , onde  $\lambda_n$  satisfaz  $\cos\sqrt{\lambda_n} - \sqrt{\lambda_n}\operatorname{sen}\sqrt{\lambda_n} = 0$   
 5.  $\phi_n(x) = \sqrt{2}e^x \operatorname{sen} n\pi x$ ;  $n = 1, 2, \dots$       6.  $a_n = \frac{2\sqrt{2}}{(2n-1)\pi}$ ;  $n = 1, 2, \dots$   
 7.  $a_n = \frac{4\sqrt{2}(-1)^{n-1}}{(2n-1)^2\pi^2}$ ;  $n = 1, 2, \dots$   
 8.  $a_n = \frac{2\sqrt{2}}{(2n-1)\pi}\{1 - \cos[(2n-1)\pi/4]\}$ ;  $n = 1, 2, \dots$   
 9.  $a_n = \frac{2\sqrt{2}\operatorname{sen}(n - \frac{1}{2})(\pi/2)}{(n - \frac{1}{2})^2\pi^2}$ ;  $n = 1, 2, \dots$

Nos Problemas de 10 a 13,  $\alpha_n = (1 + \operatorname{sen}^2\sqrt{\lambda_n})^{1/2}$  e  $\cos\sqrt{\lambda_n} - \sqrt{\lambda_n}\operatorname{sen}\sqrt{\lambda_n} = 0$ .

10.  $a_n = \frac{\sqrt{2}\operatorname{sen}\sqrt{\lambda_n}}{\sqrt{\lambda_n}\alpha_n}$ ;  $n = 1, 2, \dots$       11.  $a_n = \frac{\sqrt{2}(2\cos\sqrt{\lambda_n} - 1)}{\lambda_n\alpha_n}$ ;  $n = 1, 2, \dots$   
 12.  $a_n = \frac{\sqrt{2}(1 - \cos\sqrt{\lambda_n})}{\lambda_n\alpha_n}$ ;  $n = 1, 2, \dots$       13.  $a_n = \frac{\sqrt{2}\operatorname{sen}(\sqrt{\lambda_n}/2)}{\sqrt{\lambda_n}\alpha_n}$ ;  $n = 1, 2, \dots$   
 14. Não é autoadjunto.      15. Autoadjunto.  
 16. Não é autoadjunto.      17. Autoadjunto.  
 18. Autoadjunto.  
 21. (a) Se  $a_2 = 0$  ou  $b_2 = 0$ , não existe o termo de fronteira correspondente.  
 25. (a)  $\lambda_1 = \pi^2/L^2$ ;  $\phi_1(x) = \operatorname{sen}(\pi x/L)$   
 (b)  $\lambda_1 \cong (4,4934)^2/L^2$ ;  $\phi_1(x) = \operatorname{sen}\sqrt{\lambda_1}x - \sqrt{\lambda_1}x\cos\sqrt{\lambda_1}L$   
 (c)  $\lambda_1 = (2\pi)^2/L^2$ ;  $\phi_1(x) = 1 - \cos(2\pi x/L)$   
 26.  $\lambda_1 = \pi^2/4L^2$ ;  $\phi_1(x) = 1 - \cos(\pi x/2L)$

27. (a)  $X'' - (v/D)X' + \lambda X = 0, \quad X(0) = 0, \quad X'(L) = 0; \quad T' + \lambda DT = 0$

(e)  $c(x, t) = \sum_{n=1}^{\infty} a_n e^{-\lambda_n D t} e^{vx/2D} \sin \mu_n x$ , onde  $\lambda_n = \mu_n^2 + (v^2/4D^2)$ ;

$$a_n = \frac{4D\mu_n^2 \int_0^L e^{-vx/2D} f(x) \sin \mu_n x dx}{(2LD\mu_n^2 + v \sin^2 \mu_n L)}$$

28. (a)  $u_t + vu_x = Du_{xx}, \quad u(0, t) = 0, \quad u_x(L, t) = 0, \quad u(x, 0) = -c_0$

(b)  $u(x, t) = \sum_{n=1}^{\infty} b_n e^{-\lambda_n D t} e^{vx/2D} \sin \mu_n x$ , onde  $\lambda_n = \mu_n^2 + (v^2/4D^2)$ ;

$$b_n = \frac{8c_0 D^2 \mu_n^2 (2D\mu_n e^{-vL/2D} \cos \mu_n L + ve^{-vL/2D} \sin \mu_n L - 2D\mu_n)}{(v^2 + 4D^2\mu_n^2)(2LD\mu_n^2 + v \sin^2 \mu_n L)}$$

### Seção 11.3

1.  $y = 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin n\pi x}{(n^2\pi^2 - 2)n\pi}$

2.  $y = 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin(n - \frac{1}{2})\pi x}{[(n - \frac{1}{2})^2\pi^2 - 2](n - \frac{1}{2})^2\pi^2}$

3.  $y = -\frac{1}{4} - 4 \sum_{n=1}^{\infty} \frac{\cos(2n-1)\pi x}{[(2n-1)^2\pi^2 - 2](2n-1)^2\pi^2}$

4.  $y = 2 \sum_{n=1}^{\infty} \frac{(2 \cos \sqrt{\lambda_n} - 1) \cos \sqrt{\lambda_n} x}{\lambda_n(\lambda_n - 2)(1 + \sin^2 \sqrt{\lambda_n})}$

5.  $y = 8 \sum_{n=1}^{\infty} \frac{\sin(n\pi/2) \sin n\pi x}{(n^2\pi^2 - 2)n^2\pi^2}$

6–9. Para cada problema, a solução é

$$y = \sum_{n=1}^{\infty} \frac{c_n}{\lambda_n - \mu} \phi_n(x), \quad c_n = \int_0^1 f(x) \phi_n(x) dx, \quad \mu \neq \lambda_n,$$

onde  $\phi_n(x)$  é dada nos Problemas de 1 a 4, respectivamente, da Seção 11.2, e  $\lambda_n$  é o autovalor associado. No Problema 8, o somatório começa em  $n = 0$ .

10.  $a = -\frac{1}{2}, \quad y = \frac{1}{2\pi^2} \cos \pi x + \frac{1}{\pi^2} \left( x - \frac{1}{2} \right) + c \sin \pi x$

11. Não tem solução

12.  $a$  arbitrário,  $y = c \cos \pi x + a/\pi^2$

13.  $a = 0, \quad y = c \sin \pi x - (x/2\pi) \sin \pi x$

17.  $v(x) = a + (b-a)x$

18.  $v(x) = 1 - \frac{3}{2}x$

19.  $u(x, t) = \sqrt{2} \left[ -\frac{4c_1}{\pi^2} + \left( \frac{4c_1}{\pi^2} + \frac{1}{\sqrt{2}} \right) e^{-\pi^2 t/4} \right] \sin \frac{\pi x}{2}$   
 $- \sqrt{2} \sum_{n=2}^{\infty} \frac{4c_n}{(2n-1)^2\pi^2} [1 - e^{-(n-1/2)^2\pi^2 t}] \sin(n - \frac{1}{2})\pi x, \quad c_n = \frac{4\sqrt{2}(-1)^{n+1}}{(2n-1)^2\pi^2}, \quad n = 1, 2, \dots$

20.  $u(x, t) = \sqrt{2} \sum_{n=1}^{\infty} \left[ \frac{c_n}{\lambda_n - 1} (e^{-t} - e^{-\lambda_n t}) + \alpha_n e^{-\lambda_n t} \right] \frac{\cos \sqrt{\lambda_n} x}{(1 + \sin^2 \sqrt{\lambda_n})^{1/2}},$

$$c_n = \frac{\sqrt{2} \sin \sqrt{\lambda_n}}{\sqrt{\lambda_n}(1 + \sin^2 \sqrt{\lambda_n})^{1/2}}, \quad \alpha_n = \frac{\sqrt{2}(1 - \cos \sqrt{\lambda_n})}{\lambda_n(1 + \sin^2 \sqrt{\lambda_n})^{1/2}},$$

e  $\lambda_n$  satisfaz  $\cos \sqrt{\lambda_n} - \sqrt{\lambda_n} \sin \sqrt{\lambda_n} = 0$ .

21.  $u(x, t) = 8 \sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n^4\pi^4} (1 - e^{-n^2\pi^2 t}) \sin n\pi x$

22.  $u(x, t) = \sqrt{2} \sum_{n=1}^{\infty} \frac{c_n (e^{-t} - e^{-(n-1/2)^2\pi^2 t}) \sin(n - \frac{1}{2})\pi x}{(n - \frac{1}{2})^2\pi^2 - 1}, \quad c_n = \frac{2\sqrt{2}(2n-1)\pi + 4\sqrt{2}(-1)^n}{(2n-1)^2\pi^2}$

23. (a)  $r(x)w_t = [p(x)w_x]_x - q(x)w, \quad w(0, t) = 0, \quad w(1, t) = 0, \quad w(x, 0) = f(x) - v(x)$

24.  $u(x, t) = x^2 - 2x + 1 + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{e^{-(2n-1)^2\pi^2 t} \sin(2n-1)\pi x}{2n-1}$

25.  $u(x, t) = -\cos \pi x + e^{-9\pi^2 t/4} \cos(3\pi x/2)$

31–34. Em todos os casos a solução é  $y = \int_0^1 G(x, s) f(s) ds$ , onde  $G(x, s)$  é dado a seguir.

31.  $G(x, s) = \begin{cases} 1-x, & 0 \leq s \leq x \\ 1-s, & x \leq s \leq 1 \end{cases}$

32.  $G(x, s) = \begin{cases} s(2-x)/2, & 0 \leq s \leq x \\ x(2-s)/2, & x \leq s \leq 1 \end{cases}$

33.  $G(x, s) = \begin{cases} \cos \operatorname{sen}(1-x)/\cos 1, & 0 \leq s \leq x \\ \operatorname{sen}(1-s)\cos x/\cos 1, & x \leq s \leq 1 \end{cases}$

34.  $G(x, s) = \begin{cases} s, & 0 \leq s \leq x \\ x, & x \leq s \leq 1 \end{cases}$

### Seção 11.4

1.  $y = \sum_{n=1}^{\infty} \frac{c_n}{\lambda_n - \mu} J_0(\sqrt{\lambda_n} x), \quad c_n = \int_0^1 f(x) J_0(\sqrt{\lambda_n} x) dx / \int_0^1 x J_0^2(\sqrt{\lambda_n} x) dx,$

$\sqrt{\lambda_n}$  satisfaz  $J_0(\sqrt{\lambda}) = 0$ .

2. (c)  $y = -\frac{c_0}{\mu} + \sum_{n=1}^{\infty} \frac{c_n}{\lambda_n - \mu} J_0(\sqrt{\lambda_n} x);$

$c_0 = 2 \int_0^1 f(x) dx; \quad c_n = \int_0^1 f(x) J_0(\sqrt{\lambda_n} x) dx / \int_0^1 x J_0^2(\sqrt{\lambda_n} x) dx, \quad n = 1, 2, \dots;$

$\sqrt{\lambda_n}$  satisfaz  $J'_0(\sqrt{\lambda}) = 0$ .

3. (d)  $a_n = \int_0^1 x J_k(\sqrt{\lambda_n} x) f(x) dx / \int_0^1 x J_k^2(\sqrt{\lambda_n} x) dx$

(e)  $y = \sum_{n=1}^{\infty} \frac{c_n}{\lambda_n - \mu} J_k(\sqrt{\lambda_n} x), \quad c_n = \int_0^1 f(x) J_k(\sqrt{\lambda_n} x) dx / \int_0^1 x J_k^2(\sqrt{\lambda_n} x) dx$

4. (b)  $y = \sum_{n=1}^{\infty} \frac{c_n}{\lambda_n - \mu} P_{2n-1}(x), \quad c_n = \int_0^1 f(x) P_{2n-1}(x) dx / \int_0^1 P_{2n-1}^2(x) dx$

### Seção 11.5

1. (b)  $u(\xi, 2) = f(\xi + 1), \quad u(\xi, 0) = 0, \quad 0 \leq \xi \leq 2$   
 $u(0, \eta) = u(2, \eta) = 0, \quad 0 \leq \eta \leq 2$

2.  $u(r, t) = \sum_{n=1}^{\infty} k_n J_0(\lambda_n r) \operatorname{sen} \lambda_n a t, \quad k_n = \frac{1}{\lambda_n a} \int_0^1 r J_0(\lambda_n r) g(r) dr / \int_0^1 r J_0^2(\lambda_n r) dr$

3. Superponha a solução do Problema 2 e a do exemplo [Eq. (21)] no texto.

6.  $u(r, z) = \sum_{n=1}^{\infty} c_n e^{-\lambda_n z} J_0(\lambda_n r), \quad c_n = \int_0^1 r J_0(\lambda_n r) f(r) dr / \int_0^1 r J_0^2(\lambda_n r) dr,$   
e  $\lambda_n$  satisfaz  $J_0(\lambda) = 0$ .

7. (b)  $v(r, \theta) = \frac{1}{2} c_0 J_0(kr) + \sum_{m=1}^{\infty} J_m(kr) (b_m \operatorname{sen} m\theta + c_m \cos m\theta),$

$b_m = \frac{1}{\pi J_m(kc)} \int_0^{2\pi} f(\theta) \operatorname{sen} m\theta d\theta; \quad m = 1, 2, \dots$

$c_m = \frac{1}{\pi J_m(kc)} \int_0^{2\pi} f(\theta) \cos m\theta d\theta; \quad m = 0, 1, 2, \dots$

8.  $c_n = \int_0^1 r f(r) J_0(\lambda_n r) dr / \int_0^1 r J_0^2(\lambda_n r) dr$

10.  $u(\rho, s) = \sum_{n=0}^{\infty} c_n \rho^n P_n(s)$ , onde  $c_n = \int_{-1}^1 f(\arccos s) P_n(s) ds / \int_{-1}^1 P_n^2(s) ds;$

$P_n$  é o  $n$ -ésimo polinômio de Legendre e  $s = \cos \phi$ .

### Seção 11.6

1.  $n = 21 \quad 2. (a) b_m = (-1)^{m+1} \sqrt{2}/m\pi \quad (c) n = 20$

3. (a)  $b_m = 2\sqrt{2}(1 - \cos m\pi)/m^3\pi^3 \quad (c) n = 1$

7. (a)  $f_0(x) = 1 \quad (b) f_1(x) = \sqrt{3}(1 - 2x) \quad (c) f_2(x) = \sqrt{5}(-1 + 6x - 6x^2)$

(d)  $g_0(x) = 1, \quad g_1(x) = 2x - 1, \quad g_2(x) = 6x^2 - 6x + 1$

8.  $P_0(x) = 1, \quad P_1(x) = x, \quad P_2(x) = (3x^2 - 1)/2, \quad P_3(x) = (5x^3 - 3x)/2$