London Mathematical Society
Lecture Note Series 110

An Introduction to the Theory of Surreal Numbers

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Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
32 East 57th Street, New York, NY 10022, USA
10, Stamford Road, Oakleigh, Melbourne 3166, Australia

First published 1986

Printed in Great Britain at the University Press, Cambridge

Library of Congress cataloging in publication data

Gonshor, H.
An introduction to the theory of surreal numbers (London
Mathematical Society lecture note series; 110)
Bibliography: p.
1. Numbers, Theory of. I. Title. II. Series
QA241.G63 1986 512'.7 86-9668

British Library cataloguing in publication data

Gonshor, H.
An introduction to the theory of surreal numbers - (London
Mathematical Society lecture note series, ISSN 0076-0052; 110
1. Numbers, Theory of, 2. Logic
I. Title II. Series
512'.7 QA241

ISBN 0 521 31205 1

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PREFACE

The aim of this book is to give a systematic introduction to the theory of surreal numbers based on foundations that are familiar to most mathematicians. I feel that the surreal numbers form an exciting system which deserves to be better known and that therefore an exposition like this one is needed at present. The subject is in such a pioneering state that it appears that there are many results just on the verge of being discovered and even concepts that still are waiting to be defined.

One might claim that one should wait till the theory of surreal numbers is more fully established before publishing a book on this subject. Such a comment reminds me of the classic joke about the person who is afraid of drowning and has vowed never to step into water until he has learned how to swim. In fact, the time is ripe for such a book and furthermore the book itself should contribute to developing the subject with the help of creative readers.

The subject has suffered so far from isolation with pockets of people in scattered parts of the world working on those facets of the subject that interest them. I hope that this book will play a role in eliminating this isolation and bringing together the mathematicians interested in surreal numbers.

The book is thus a reflection of my own personal interest. For example, Martin Kruskal has developed the theory of exponentiation from a somewhat different point of view and carried it in different directions from the presentation in this book. Also, I recently received correspondence from Norman Alling who has recently done work on a facet of the theory of surreal numbers not discussed in the book. With greater communication all this and more could play a role in a future edition.

The basic material is found in chapters 2 through 5. The later chapters are more original and more specialized. Although room for future improvement exists everywhere, chapters 7 and 8 are in an especially pioneering position: this is where the greatest opportunity seems to exist for knowledgeable readers to obtain new results.

ACKNOWLEDGEMENTS

I would like to thank the following people for their help in connection with the manuscript.

First, there is Professor Larry Corwin who took time from his very busy schedule to do a great deal of proofreading. He is responsible for many improvements in the exposition throughout the manuscript. On the other hand, I take full responsibility for any faults in the exposition which still remain. Professor Joe D'Atri and Jim Maloney, a graduate student, have also helped with some proofreading. Also, I should mention Professor Barbara Osofsky who much earlier had read a preliminary draft of chapters one and two and made many valuable suggestions.

Finally, I mention the contribution of two secretaries of the Rutgers Mathematics Department. Mary Anne Jablonski, the co-ordinator, took care of numerous technical details, and Adelaide Boulle did an excellent job of turning my handwritten draft into typescript.

LONDON MATHEMATICAL SOCIETY
LECTURE NOTE SERIES

An Introduction to the Theory of Surreal Numbers

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The surreal numbers form a system which includes both ordinary real numbers and the ordinals. Since their introduction by J. H. Conway, the theory of surreal numbers has seen a rapid development revealing many natural and exciting properties. These notes provide a formal introduction to the theory. The topics covered include exponentiation and generalised ε-numbers.

Professor Gonshor has presented here the basic results and key ideas in a clear and lucid style. The subject is still new and he is able to lead the reader through to some of the outstanding problems in the field.

The prerequisites are minimal, so undergraduates, research students and professionals in mathematics will find this a readily-accessible introduction.

ISBN 0-521-31205-1
9 780521 312059