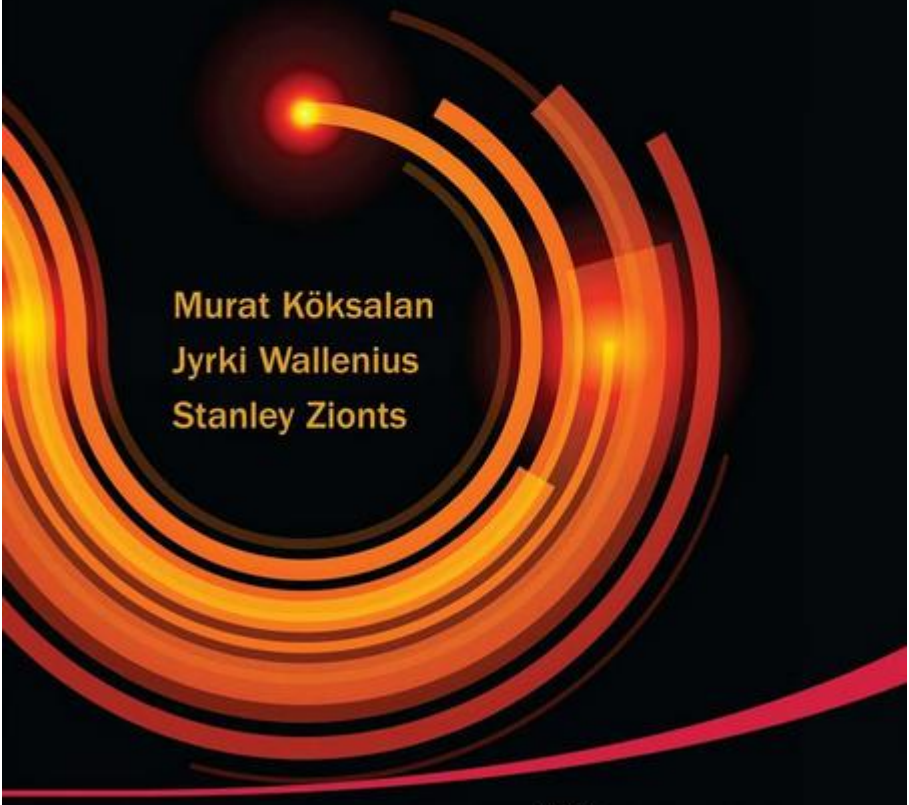
Multiple Criteria Decision Making

From Early History to the 21st Century



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MULTIPLE CRITERIA DECISION MAKING
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Contents

Preface

What led to the writing of this book?

The field of Multiple Criteria Decision Making (MCDM) can be said to be both old and new, depending on one's frame of reference. It is old because people have always had to trade off objectives in making decisions. Possibly the first recorded discussion of trade-offs in making decisions and consideration of multiple objectives is the American statesman Benjamin Franklin's way of deciding his position on important decisions. Franklin lived during the 1700s. Many modern researchers have considered MCDM problems. The problem may be represented as an evaluation problem, where the decision maker chooses among a finite set of discrete alternatives; or as a design problem, where the set of decision alternatives is described with a mathematical model. Among the research, the work of Abraham Charnes and William Cooper on goal programming in the late 1950s was a major stimulus to the later explosion of MCDM work. Since then, more than 15,000 papers and numerous books have been written, all of which can be regarded as MCDM contributions from the 1960s to the present time. Hence MCDM is clearly an important subfield of Management Science or Operations Research, or as a matter of fact, an important field in its own right. The MCDM field has experienced exponential growth in terms of the number of publications as well as the number of citations. The roots of the field are relatively old, extending to research of classical economists and mathematicians. Recent foundations of MCDM were developed in the 1950s and 1960s. The 1970s was an important decade in which many seminal contributions were produced, with the field maturing during the 1980s. MCDM experienced accelerated development during the early 90s and seems to have continued its

exponential growth. As an outcome of the growth many subfields have emerged. A recent subfield is Evolutionary Multiobjective Optimization (EMO), in which there have been many active researchers. Interestingly, MCDM has penetrated and continues to penetrate many engineering fields, as well as medicine.

The initials MCDM, of course, stand for Multiple Criteria Decision Making. A paper by Stan Zionts, entitled "MCDM — If not a Roman Numeral, then What?" published in 1979, helped make MCDM an accepted abbreviation for the field.

The book by Saul Gass and Arjang Assad, *An Annotated Timeline of Operations Research: An Informal History* published in 2005, provided an impetus for writing this book. The Gass–Assad volume explored the timeline of Operations Research, as well as many of its developments. It also includes biographies of many contributors to the field. Though some MCDM contributors and contributions are included in their book, many are not or are just very briefly mentioned. Therefore we thought of producing a book devoted only to MCDM, its contributions and contributors.

We consider MCDM to consist of many subfields, such as Decision Analysis, Goal Programming, work of the "French School," which includes outranking relations, Multiple Objective Mathematical Programming, Fuzzy Set Theory, the Analytic Hierarchy Process (AHP), and Evolutionary Multiobjective Optimization (EMO). We have tried to include all important subfields, contributions, and contributors in compiling this volume. Any omissions are ours, and we apologize in advance for them.

The criteria we used to select contributions and contributors for the volume came from various sources. We used citation statistics from the ISI Web of Science and Google Scholar, a powerful search engine provided under the Google umbrella. We included contributions and contributors we felt were important to the field, well known scholars and their contributions. We were sure to include recipients of our society's (the International Society on Multiple Criteria Decision Making) awards and their main contributions (there have been 34 awards to date — given at meetings of the society).

We decided to organize the book chronologically. We begin with the early history of MCDM, which covers the roots of MCDM through the

1960s. Then we proceed decade by decade, with one chapter each covering the 1970s, the 1980s, as well as the 1990s and beyond. The impact of recent developments have not yet been fully observed. Therefore, we included the developments in the 2000s together with the 1990s and tried to concentrate on trends observed during that decade. We also mention highlights of the twenty conferences that our society has had. The most recent MCDM conference took place in Chengdu, Sichuan Province, China in June, 2009; the next one is scheduled for Jyväskylä, Finland in June 2011. Each meeting has been unique: we have had conferences in more than a dozen countries around the world. Some have been lavish; others have been less so. Yet each has made its mark in its own way, both in terms of research presented and in terms of the culture of the host area and the hosts.

We did not however, adhere strictly to the chronological order. If a topic is covered only in a certain section, we mention the developments from other decades in that section as well. In some other cases, we decided to discuss topics across different decades to maintain continuity.

We have included brief biographies with pictures of major contributors, which allow for interesting stories and lives to emerge. We obtained help for the biographies and pictures from contributors to the field where possible but used other sources as well. We have also, when possible, included personal aspects of the contributors' lives, emphasizing the human side of the researchers.

One of our inspirations for this book project was Murat Köksalan's presentations in his classes, which summarize major developments in the field together with the pictures of prominent MCDM scholars. We tried to produce a proper mixture of pictures and history.

Why us? Why did we write this book? What are our qualifications for writing such a book? We are all seasoned MCDM scholars, with a collective memory extending back to the 60s. **Murat Köksalan**, a graduate of the State University of New York (SUNY) Buffalo (under Mark Karwan and Stan Zionts) served many years as a member of the Executive Committee of the International Society on Multiple Criteria Decision Making. He was a past chairperson of the Society's Awards Committee and the chairperson of the organizing committee for the 15th International MCDM Conference held in Ankara, Turkey. He is the

founding president of the INFORMS Section on MCDM. His research has included problems of Multiple Criteria Decision Making, multiobjective combinatorial optimization, decision support, heuristic search and evolutionary algorithms, among others. He was awarded the Gold Medal of the International Society on Multiple Criteria Decision Making in 2006. **Jyrki Wallenius** is the current President of the International Society on Multiple Criteria Decision Making. His research has covered problems of Multiple Criteria Decision Making, negotiation analysis, behavioral decision making, decision support, and online auctions. Wallenius is a former editor of the European Journal of Operational Research. He received the Edgeworth–Pareto award from the International Society on Multiple Criteria Decision Making in 1994. **Stanley Zionts** is Distinguished Professor Emeritus at the State University of New York at Buffalo, where he served on the faculty from 1967 until 2005. He served as Professor of Management at the European Institute for Advanced Studies in Management in Brussels from 1973 to 1975, where Wallenius was his student. Zionts was the founder and first president of the Special Interest Group on MCDM, a predecessor of the International Society on Multiple Criteria Decision Making. Under his leadership the society grew into an international organization with membership of over 1,000 scholars in about 80 different countries. He was the organizer of several international MCDM conferences. His research has included various aspects of linear and integer programming, Multiple Criteria Decision Making, negotiation analysis, decision support, and finance. Zionts was awarded the Gold Medal as well as the Presidential Service Award of the International Society on Multiple Criteria Decision Making in 1992.

Many of the pictures are from the authors' private sources or the contributors themselves. In addition, we have obtained permission to reuse pictures from various sources, including Princeton University, The Archives of the Mathematisches Forschungsinstitut Oberwolfach, and Aalto University School of Economics. Some pictures are in the public domain. Many people have helped us with the compilation of information in this volume. First, we thank the contributors to the field, with whom we have been in contact. In addition, we wish to thank the many other people who have helped us with information or pictures. This

includes Günter Fandel, Saul Gass, Pekka Korhonen, and Ralph Steuer. Saul Gass deserves a special thank you. Besides furnishing us with several pictures, he provided us with valuable advice regarding the use of pictures.

We have enjoyed preparing and writing this book. It has been both fun and educational. We hope that readers, whether MCDM or related researchers, graduate students, emeritus professors, or just laypersons, enjoy reading the book as much as we have enjoyed writing it.

Chapter 1

The Early History of MCDM

The practice of decision making is ancient. Yet, the origins of the field are somewhat obscure. We can, however, separately trace the origins of decision analysis/utility theory and the origins of multiple objective mathematical programming. The earliest known reference relating to Multiple Criteria Decision Making (although not using that name) can be traced to Benjamin Franklin (1706–1790), the American statesman, who allegedly had a simple paper system for deciding his position on an important issue. He explained his procedure in a letter to a friend, Joseph Priestly. Take a sheet of paper. On one side write the arguments in favor of a decision; on the other side the arguments against. Cross out arguments on each side of the paper that are relatively of equal importance. Franklin did in fact talk about weights, though he did not describe any actual use of weights. When all the arguments on one side have been crossed out, the side with arguments not crossed out is the side of the argument that should be supported. Franklin supposedly used this in making important decisions.

Marie-Jean-Antoine-Nicolas de Caritat (better known with his title, Marquis de Condorcet, 1743–1794), a French mathematician and political scientist, was a pioneer in applying mathematics to the social sciences, in particular to elections. He wrote the famous *Essay on the Application of Analysis to the Probability of Majority Decisions* in 1785. This paper described Condorcet's jury theorem, Condorcet's paradox, and the so-called Condorcet method. Condorcet's paradox is perhaps the most famous of his results. It states that majority preferences may become intransitive even though individual preferences are transitive. He disagreed with a contemporary scholar, another French mathematician and political scientist, Jean-Charles de Borda (1733–1794), who advocated the use of summed rankings.

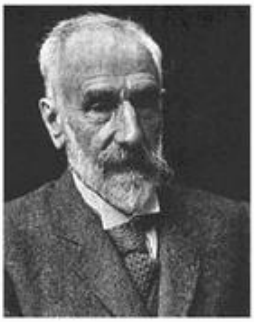
(Left) Benjamin Franklin (1706–1790): American statesman, author, inventor. Inspired MCDM scholars by his "Moral Algebra."

(Right) Georg Cantor (1845–1918): Mathematician whose work impacted the mathematical foundations of multiobjective optimization.

Marquis de Condorcet

Georg Cantor (1845–1918) was a German mathematician born in St. Petersburg, Russia. He is known to be the creator of set theory. He made many other fundamental contributions to mathematics. These contributions are also the foundations of the mathematical concepts used in MCDM. Since 1992, the International Society on Multiple Criteria Decision Making has been giving out Georg Cantor awards.

Francis Edgeworth (1845–1926) was an influential person in the development of neoclassical economics. He was the first to apply certain



(Left) Francis Ysidro Edgeworth (1845–1926)
(Right) Vilfredo Pareto (1848–1923) developed main concepts used in MCDM.

formal mathematical concepts to decision making. He developed the foundations of utility theory, introducing the notion of an indifference curve and the famous Edgeworth box. He was appointed Professor of Economics at King's College London in 1888, and later Professor of Political Economy at Oxford University. An Edgeworth box is a way of representing various distributions of resources. Edgeworth described the box in his famous book: *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences* (1881).

More recently, the economist Vilfredo Pareto (1848–1923), born in Paris to Italian expatriates, was the first to mathematically study the aggregation of conflicting criteria into a single composite index. He was also the first to introduce the concept of efficiency (which became known as Pareto-optimality), one of the key concepts of economics and modern MCDM theory. A Pareto-optimal allocation of resources is achieved when it is not possible to make anyone better off without making at least one other person worse off. Pareto graduated from the Polytechnic Institute of Turin in 1869. Throughout his life, Pareto actively criticized the Italian government's economic policies, despite not studying economics seriously until he was over forty years old. In 1893, he succeeded Leon Walras as Professor of Economics at the University of Lausanne. His main

Following the traditions in optimal control, M.E. Salukvadze[1] published three papers in *Automation and Remote Control*, two in 1971 and one in 1972, all of which are of historical significance. They represent one of the first attempts to employ the "ideal point" to scalarize problems having multiple objectives. Designed to solve control problems with multiple (vector) functionals, the method minimizes the Euclidean distance between the ideal trajectory and the set of feasible trajectories. No interaction with the decision maker takes place. Po-Lung Yu and George Leitmann criticized Salukvadze's choice of the metric as arbitrary and presented a more general metric in a *JOTA* 1974 publication. Another important early Soviet paper was A. Lotov's "Numerical Method of Constructing Attainability Sets for a Linear Control System," published in 1972. This paper essentially laid the foundation for Lotov's reachable set method, which made projections of the constraint set onto the subspace of objective functions. The method relies on the theory of linear inequalities to generate the projections. V.A. Bushenkov and A. Lotov provided extensions and extensively published on the idea in the 80s.

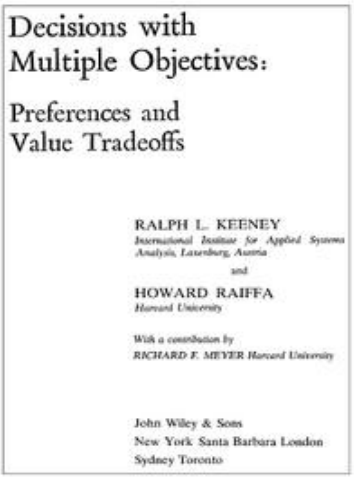
Ralph E. Steuer published his first paper jointly with his thesis supervisor James Evans, in *Mathematical Programming* (Evans and Steuer, 1973). The title of the paper was "A Revised Simplex Method for Linear Multiple Objective Programs." For linear multiple objective problems, a necessary and sufficient condition for a point to be efficient was employed to develop a revised simplex algorithm for the enumeration of all efficient extreme points. The algorithm was the first of its kind. Several options within this algorithm were tested on a variety of problems. This was followed by the development of the ADBASE program to generate all

[1] For interesting papers on Soviet research on MCDM, see V.M. Ozernoy's survey article published in 1988, E. Lieberman's overview in *Management Science* in 1991, and E. Lieberman's book published in 1991. The publications review Soviet MCDM research since 1971. Theoretical methodology research dominated the Soviet literature. Interestingly, the interactive approaches seemed most popular. The overall focus was on solving "difficult" problems (with irregular feasible regions) commonly found in engineering. Several of the approaches bear semblance to approaches presented in the west, although there were also distinctively different ideas. We mention what we believe to be the most innovative ideas. Stancu-Minasian compiled a bibliography of 421 selected works related to MCDM in 1975.

have the decision maker move along the efficient surface of that polyhedron via iteratively solving a so-called "lambda problem."[3] Before the ideas took shape, they also explored what they called a naive method, where the decision maker would simply move from one efficient extreme point solution to another. For large scale problems, which Zionts and Wallenius had in mind, this would, however, take an eternity. Hence it was important to make the algorithm efficient in terms of numbers of questions asked. Part of the algorithm was the "efficiency routine" to test for adjacent efficient solutions, which the authors published separately in *Operations Research* in 1980. Along with the Geoffrion, Dyer and Feinberg method, it became a standard reference. They started a stream of research in interactive multiple objective mathematical programming that is still popular today. Their first working paper was called "A Simplex-Type Interactive Programming Method for Solving the Multiple Criteria Problem," October 1973 (EIASM Working Paper #44), emphasizing the fundamental idea of extending the simplex method to multiple objective linear programming. The authors were hopeful that companies would start using their method, since many of them already used linear programming, and it would be a relatively small step to extend it to multiple objective linear programming. The real world was, however, not that simple!

Ralph Keeney and Howard Raiffa published their book "Decisions with Multiple Objectives: Preferences and Value Tradeoffs" in 1976. According to the authors, they worked on the book for about six years. The book summarized much of the authors' research dating from 1968 to 1975. An important paper (covered in their book) was Keeney's *Management Science* paper published in 1972, discussing utility functions for multiattributed consequences. The Keeney–Raiffa book was instrumental in establishing the theory of multiattribute value and utility measurement as a discipline. It became a standard reference and textbook for many generations of graduate students in decision analysis and MCDM. In a thorough treatment, they covered trade-offs under certainty and under uncertainty, both the two-attribute and the n-attribute cases. They also included a chapter on the (now) classic application concerning

[3] The idea of the "lambda problem" has been applied numerous times in different contexts. A recent application is to internet search. See Asim Roy *et al.* (2008).


Decisions with
Multiple Objectives:

Preferences and
Value Tradeoffs

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Inside cover of R. Keeney and H. Raiffa's 1976 book.

the airport development options for Mexico City, originally published by de Neufville and Keeney in 1974. Keeney and Raiffa finished their book while at IIASA in Laxenburg, Austria. As the authors state, "this is a big book and not all of it has to be read!" Interestingly, their 1976 book contained the rudiments of a sequel by Ralph Keeney in 1992 called *Value Focused Thinking*.

Lawrence D. Phillips and Detlof von Winterfeldt have reflected on the contributions of Ward Edwards to decision research after Edwards' death (*Advances in Decision Analysis*, edited by Ward Edwards, Ralph F. Miles Jr., and Detlof von Winterfeldt, 2007). Ward Edwards apparently liked multiattribute utility theory. However, he thought that the Keeney and Raiffa approach was too difficult to use in practice. Accordingly, he created a simpler version of this method — which later came to be known as SMART — the simple multiattribute rating technique, which he published in two papers (1971; 1977). The SMART method attracted many

versions of SMART and applied them to social problems. One quite remarkable application was an evaluation of school desegregation plans for the Los Angeles Unified School District.

Peter Fishburn made many fundamental contributions to the theory of social choice and utility theory. During the 70s and 80s, he continued his research on utility theory, at times dealing with two-attribute and multi-attribute situations. During the 1980s, Peter Fishburn published several books, including *The Foundations of Expected Utility* (1982) and *Nonlinear Preference and Utility Theory* (1988). Many of his contributions are fundamental and embedded in multiattribute utility theory, such as his research on lexicographic preferences, independent preferences, and nonlinear preferences. As mentioned in the citation of his John von Neumann Theory Prize, Fishburn has also made significant contributions to group decision making, including decisions based on voting processes. Some of these contributions can be found in his book, *The Theory of Social Choice* (1973), as well as the numerous articles on social choice functions, majority choice, and Arrow's impossibility theorem.

Andrzej Wierzbicki, a Polish mathematician with a background in optimal control theory, published the first two papers dealing with the achievement scalarizing function in 1975 and 1977. The 1979 Königswinter Conference Proceedings paper from 1980 reviews his early thoughts. In subsequent papers, Wierzbicki further extended and refined his ideas. The use of the achievement scalarizing function effectively means optimization of a "value function," which is defined in terms of a reference point given by the decision maker. This is not equivalent to minimizing the distance (or a norm) to the reference point, commonly used in goal programming. A remarkable feature of the achievement scalarizing function is that it can be used to project both infeasible and feasible points to the efficient frontier. By adding an auxiliary term to the achievement scalarizing function, one can avoid the generation of inefficient solutions among the set of so-called weakly efficient solutions. Furthermore, it is possible to generate any efficient solution by formulating an appropriate achievement scalarizing function. The definition of the achievement scalarizing function was a breakthrough and its use has become commonplace in interactive methods. Wierzbicki was active in developing

computerized decision support systems based on the idea of the achievement scalarizing function.

Jared Cohon and Yacov Haimes independently pioneered applications of MCDM models to water resource management. J.L. Cohon and D.H. Marks published a paper "Multiobjective Screening Models and Water Resource Investment" in 1973. Environment and sustainable development reflected the interests of Jared Cohon, while serving as Dean of the School of Forestry and Environmental Studies at Yale University. Moreover, Jared Cohon published a significant book, *Multiobjective Programming and Planning* by Academic Press in 1978, taking a broad view of multiple objective programming, yet emphasizing the methods most useful for continuous problems. Y. Haimes and W. Hall published in 1974, a paper developing the surrogate worth trade-off method, applied to water resource management. Y. Haimes, W. Hall and H. Friedman published a book dealing with multiobjective optimization in water resources systems in 1975.

According to Forman and Gass, Thomas L. Saaty, while teaching at the Wharton School in the 70s, was troubled by the communication difficulties he had observed between the scientists and lawyers concerning priority setting and decision making (Forman and Gass, 2001). Saaty was motivated to develop a simple way to help lay people make complex decisions. The result was the analytic hierarchy process (AHP). The first publications appeared in 1977 ("A Scaling Method for Priorities in Hierarchical Structures," *Journal of Mathematical Psychology*; and "The Sudan Transport Study," *Interfaces*). His first book on the AHP was published by McGraw Hill in 1980 with the title *Analytic Hierarchy Process*. According to Saaty, AHP is a method to derive ratio scales from pair-wise comparisons. The input can be obtained from objective measurements such as prices, weights etc., or from subjective measurements such as feelings and preferences. AHP allows some inconsistency in judgments. The degree of inconsistency is reported by an inconsistency index. The weights are derived from the principal eigenvectors. Over a period of 30 years, the AHP has become one of the most celebrated MCDM tools for practitioners. It is being widely used by corporations and governments all over the world. Several commercial software implementations exist, the first being EXPERT CHOICE, originally developed in 1983.

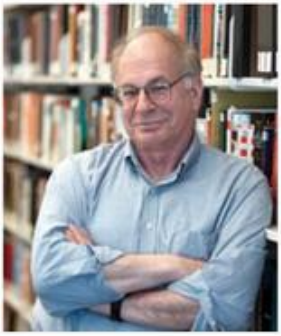
James Dyer and Rakesh Sarin published "The Measurable Multiattribute Value Function" paper in *Operations Research* in 1979. Such functions are based on the concept of a "preference difference" between alternatives and provide an interval scale of measurement for preferences under certainty. The authors present conditions for additive, multiplicative, and more complex forms of the measurable multiattribute value function.

Building on earlier ideas of Farrell and developed in Rhodes' doctoral dissertation, data envelopment analysis (DEA) was developed by A. Charnes, W.W. Cooper and E. Rhodes in their *EJOR* article "Measuring the Efficiency of Decision Making Units" in 1978.

During the 70s, conjoint analysis received attention as an approach to measuring consumers' trade-offs among products and services possessing multiple attributes. The focus was on explaining and predicting consumer choices rather than aiding consumers in their choice problems. The models were often referred to as "attitude models." See P. Green and V.R. Rao's review paper on conjoint analysis in 1971, V. Srinivasan and A.D. Shocker's paper dealing with the weight estimation problem (Srinivasan and Shocker, 1973b), and the paper by W. Wilkie and E. Pessemier about multiattribute attitude models in 1973. The reader is also referred to the interesting linear programming-based technique for preference analysis by Srinivasan and Shocker (1973a).

In 1979, Daniel Kahneman and Amos Tversky published their celebrated prospect theory paper, as an alternative to the expected utility model. Prospect theory could explain many of the paradoxes of the classical utility theory, among others, the famous Allais paradox. One of the key ideas was that people make choices with respect to a reference point. They consider gains and losses with respect to such a reference point. Moreover, people react more strongly to negative than to positive stimuli of the same magnitude, exhibiting loss aversion. The original model was extended to riskless choice in Tversky and Kahneman (1991).[4] Although

[4] Independently from Tversky and Kahneman, Pekka Korhonen, Herbert Moskowitz and Jyrki Wallenius considered the application of prospect theory to riskless choice in their *Annals of Operations Research* paper in 1990. Their first versions date back to 1986 and 1987. See Korhonen, Moskowitz, and Wallenius (1992).



(Left) Daniel Kahneman

Courtesy: Wikimedia Commons, http://en.wikipedia.org/wiki/List_of_Nobel_laureates_in_Economics.

(Right) Amos Tversky

intended as a descriptive model of choice, the key ideas have been applied to normative MCDM models. Daniel Kahneman received the Nobel Prize in Economics in 2002 for his contributions to the psychology of economics. Amos Tversky died six years earlier; otherwise he would almost certainly have shared the prize with Kahneman.

Po-Lung Yu published the fundamental ideas of his habitual domain theory in the Königswinter Conference Proceedings in 1980. The paper was based on a 1979 working paper, which he wrote at the School of Business, University of Kansas. Yu argues that people develop a fairly stable set of ways of thinking and making judgments, which he calls a habitual domain. The theory essentially integrates findings from psychology, mathematics, and common sense. Yu discusses the formation of habitual domains and how one can expand one's habitual domain. The paper concludes by discussing some applications. According to the author, a number of state variables are used to describe human physiological conditions, the social situation, and individual's goals. Their values are constantly monitored. When the current value falls significantly below its goal value, a "charge" (tension) is produced. The purpose of "attention" is

From left: Milan Zeleny, Po-Lung Yu, and Jonathan Kornbluth at the Manchester Conference, 1988.

Po-Lung Yu and Herb Moskowitz, Helsinki, 1988. They gave joint talks on Habitual Domains. Moskowitz is Distinguished Professor Emeritus at Purdue University.

to release the "charges" efficiently. Over thirty years, Po-Lung Yu has extensively written about his ideas related to "Habitual Domains" and presented seminars to business people about how they can expand their habitual domains. Po-Lung's ideas have found widespread acceptance, particularly in Asia. *Forming Winning Strategies: An Integrated Theory of Habitual Domains*, published by Springer in 1990, is a popular book. Po-Lung Yu has also given many tutorial presentations in scientific conferences about "Habitual Domains." Younger audiences associate Po-Lung Yu's name with "Habitual Domains," forgetting his past in optimal control and game theory.

Chapter 3

MCDM Developments in the 1980s

The MCDM field matured during the 1980s. Many important books were published covering developments in MCDM.[1] We highlight several of the books here. Steuer's book *Multiple Criteria Optimization* provided an influential coverage of multiobjective mathematical programming techniques. Von Winterfeldt and Edwards' excellent book *Decision Analysis and Behavioral Research* has not yet had a desired impact on multiple criteria research, despite calls for additional behavioral realism in models by several scholars. Spronk's book *Interactive Multiple Goal Programming* was one of the earliest to draw attention to applications of MCDM models in finance. Zeleny's book *Multiple Criteria Decision Making* provided a broad coverage of the field, always in search of new paradigms. Saaty, in his 1980 book, focused on his newly developed analytic hierarchy process (AHP) methodology, which continued to thrive during the 1980s and beyond.

Review articles on MCDM began to appear[2]: Roy and Vincke, and later Roy and Vanderpooten, covered developments of the French school.

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[1] Saaty, *The Analytic Hierarchy Process*, 1980; Spronk, *Interactive Multiple Goal Programming*, 1981; Zeleny, *Multiple Criteria Decision Making*, 1982; Goicoechea, Hansen, and Duckstein, *Multiobjective Decision Analysis with Engineering and Business Applications*, 1982; Chankong and Haimes, *Multiobjective Decision Making: Theory and Methodology*, 1983; Sawaragi, Nakayama, and Tanino, *Theory of Multiobjective Optimization*, 1985; Yu, *Multiple Criteria Decision Making*, 1985; Steuer, *Multiple Criteria Optimization*, 1986; von Winterfeldt and Edwards, *Decision Analysis and Behavioral Research*, 1986; and Vincke, *Multicriteria Decision-Aid*, 1992.

[2] Roy and Vincke, *EJOR*, 1981; Roy and Vanderpooten, *JMCDA*, 1996; Farquhar, 1983; Evans, *Man. Sci.*, 1984; Romero, *EJOR*, 1986; Ozernoy, *NRL*, 1988; Aksoy, *Man. Res. News*, 1990; Lieberman, *Man. Sci.*, 1991; Shin and Ravindran, *EJOR*, 1992; Stewart, *OMEGA*, 1992; Dyer *et al.*, *Man. Sci.* 1992.