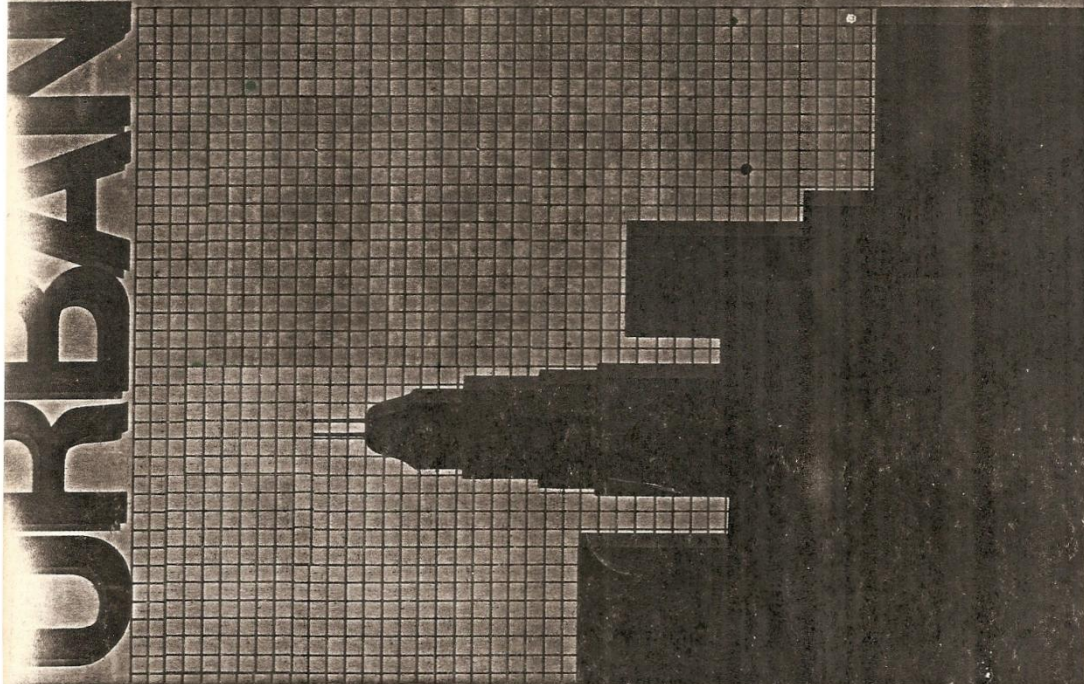


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Prof: Altamir A. R. Araldi

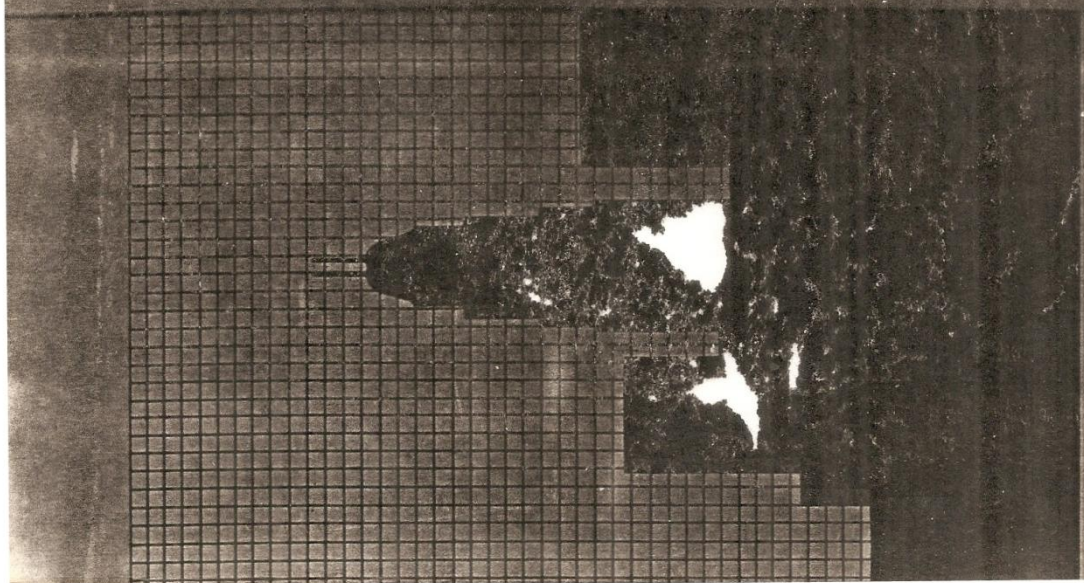
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## OPERATIONS RESEARCH



Odoni

## URBAN OPERATIONS RESEARCH



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*To Alvin Drake and Alfred Blumstein  
who stimulated and encouraged our early work  
in public systems*

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