Hoel
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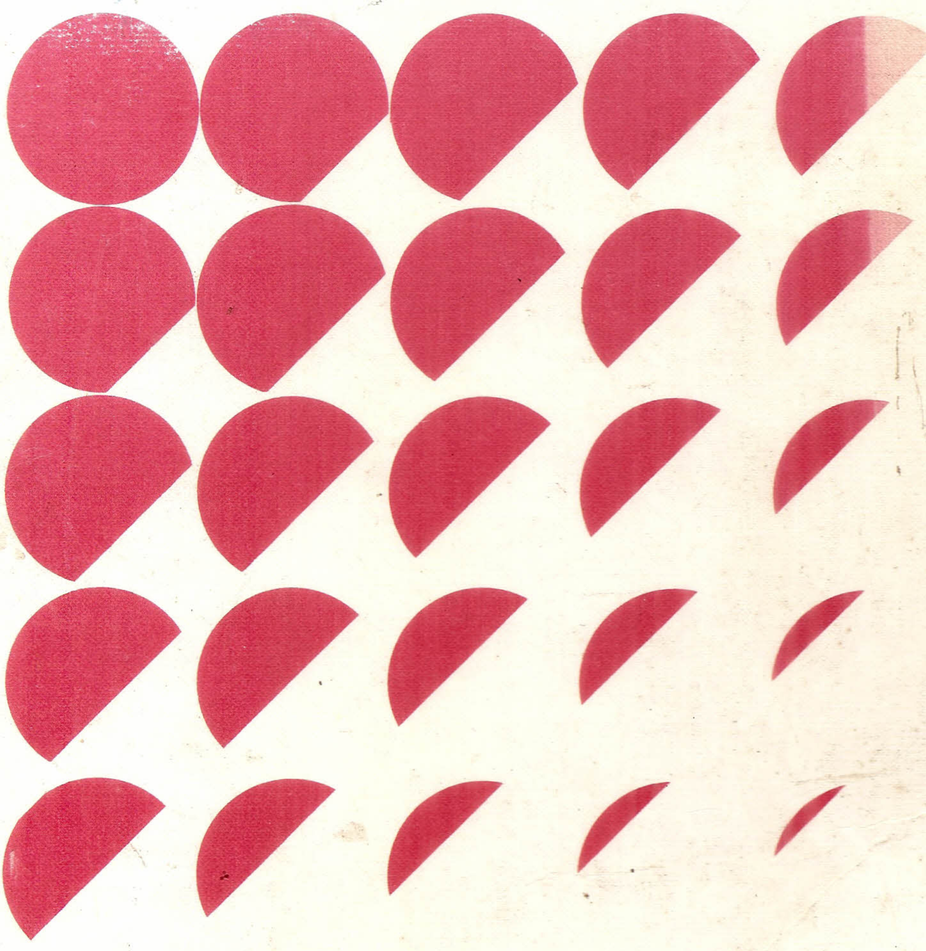
Introduction to Stochastic Processes



The Houghton Mifflin Series in Statistics under the Editorship of Herman Chernoff

LEO BREIMAN

Probability and Stochastic Processes: With a View Toward Applications

Statistics: With a View Toward Applications

PAUL G. HOEL, SIDNEY C. PORT, AND CHARLES J. STONE

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Introduction to Statistical Theory

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Introduction to Stochastic Processes

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General Preface

This three-volume series grew out of a three-quarter course in probability, statistics, and stochastic processes taught for a number of years at UCLA. We felt a need for a series of books that would treat these subjects in a way that is well coordinated, but which would also give adequate emphasis to each subject as being interesting and useful on its own merits.

The first volume, *Introduction to Probability Theory*, presents the fundamental ideas of probability theory and also prepares the student both for courses in statistics and for further study in probability theory, including stochastic processes.

The second volume, *Introduction to Statistical Theory*, develops the basic theory of mathematical statistics in a systematic, unified manner. Together, the first two volumes contain the material that is often covered in a two-semester course in mathematical statistics.

The third volume, *Introduction to Stochastic Processes*, treats Markov chains, Poisson processes, birth and death processes, Gaussian processes, Brownian motion, and processes defined in terms of Brownian motion by means of elementary stochastic differential equations.

Preface

In recent years there has been an ever increasing interest in the study of systems which vary in time in a random manner. Mathematical models of such systems are known as stochastic processes. In this book we present an elementary account of some of the important topics in the theory of such processes. We have tried to select topics that are conceptually interesting and that have found fruitful application in various branches of science and technology.

A *stochastic process* can be defined quite generally as any collection of random variables $X(t)$, $t \in T$, defined on a common probability space, where T is a subset of $(-\infty, \infty)$ and is thought of as the time parameter set. The process is called a *continuous parameter process* if T is an interval having positive length and a *discrete parameter process* if T is a subset of the integers. If the random variables $X(t)$ all take on values from the fixed set $\mathscr{S}$, then $\mathscr{S}$ is called the *state space* of the process.

Many stochastic processes of theoretical and applied interest possess the property that, given the present state of the process, the past history does not affect conditional probabilities of events defined in terms of the future. Such processes are called *Markov processes.* In Chapters 1 and 2 we study *Markov chains,* which are discrete parameter Markov processes whose state space is finite or countably infinite. In Chapter 3 we study the corresponding continuous parameter processes, with the "Poisson process" as a special case.

In Chapters 4–6 we discuss continuous parameter processes whose state space is typically the real line. In Chapter 4 we introduce *Gaussian processes,* which are characterized by the property that every linear combination involving a finite number of the random variables $X(t)$, $t \in T$, is normally distributed. As an important special case, we discuss the *Wiener process,* which arises as a mathematical model for the physical phenomenon known as "Brownian motion."

In Chapter 5 we discuss integration and differentiation of stochastic processes. There we also use the Wiener process to give a mathematical model for "white noise."

In Chapter 6 we discuss solutions to nonhomogeneous ordinary differential equations having constant coefficients whose right-hand side is either a stochastic process or white noise. We also discuss estimation problems involving stochastic processes, and briefly consider the "spectral distribution" of a process.

This text has been designed for a one-semester course in stochastic processes. Written in close conjunction with *Introduction to Probability Theory*, the first volume of our three-volume series, it assumes that the student is acquainted with the material covered in a one-semester course in probability for which elementary calculus is a prerequisite.

Some of the proofs in Chapters 1 and 2 are somewhat more difficult than the rest of the text, and they appear in appendices to these chapters. These proofs and the starred material in Section 2.6 probably should be omitted or discussed only briefly in an elementary course.

An instructor using this text in a one-quarter course will probably not have time to cover the entire text. He may wish to cover the first three chapters thoroughly and the remainder as time permits, perhaps discussing those topics in the last three chapters that involve the Wiener process. Another option, however, is to emphasize continuous parameter processes by omitting or skimming Chapters 1 and 2 and concentrating on Chapters 3–6. (For example, the instructor could skip Sections 1.6.1, 1.6.2, 1.9, 2.2.2, 2.5.1, 2.6.1, and 2.8.) With some aid from the instructor, the student should be able to read Chapter 3 without having studied the first two chapters thoroughly. Chapters 4–6 are independent of the first two chapters and depend on Chapter 3 only in minor ways, mainly in that the Poisson process introduced in Chapter 3 is used in examples in the later chapters. The properties of the Poisson process that are needed later are summarized in Chapter 4 and can be regarded as axioms for the Poisson process.

The authors wish to thank the UCLA students who tolerated preliminary versions of this text and whose comments resulted in numerous improvements. Mr. Luis Gorostiza obtained the answers to the exercises and also made many suggestions that resulted in significant improvements. Finally, we wish to thank Mrs. Ruth Goldstein for her excellent typing.

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