Metodologias Multicritérios de Apoio à Decisão

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Applications of the MACBETH approach in the framework of an additive aggregation model

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ABSTRACT

In Measurement Theory terminology, MACBETH is an interactive approach for mapping into a real scale the various degrees to which the elements of a finite set possess a property *P*. The originality of MACBETH's questioning procedure is the possibility of establishing a constructive path towards cardinal measurement in both quantitative and substantive meaningful terms, avoiding the operational problems recognised as a weakness of other procedures. The use of the notion of semantic absolute judgements plays a key role here, and the simplicity, interactivity and constructiveness of our approach insert it in the modern paradigms of decision aid.

This article illustrates the usefulness of MACBETH as a tool to facilitate decision support, by describing its application in two real public decision situations where we acted as facilitators. Although the cases correspond to quite different decision contexts and problematics, and our interventions have addressed diverse issues, they have in common the fact that we conducted both evaluation processes in the framework of an additive multicriteria aggregation procedure. In the first case MACBETH has been used to support the construction of (local) cardinal value functions, while the second illustrates how our approach can serve as a weighting procedure to determine the scaling constants of an additive aggregation model.

KEY WORDS Multicriteria Analysis Cardinal value functions Scaling constants Applications

1. INTRODUCTION

Structuring and evaluating are essential phases in applications of a decision-aid methodology. Both contribute to robust decision-making. Structuring provides the actors involved in a problematic situation with a common language for debate and learning, and with clear information about the plausible impacts of potential actions on the fundamental points of view which make explicit the actors' value systems. Thus, structuring offers a sound basis for the identification of decision opportunities, for the construction of new alternatives, and for evaluating potential actions. In an evaluation aid process, the facilitator takes steps towards the formal development of an *overall evaluation model*. In a multicriteria methodological framework, it can be split to:

(1) the construction of a *criterion* for each fundamental point of view, that is, an evaluation model that formally represents the *local attractiveness* of potential actions for some evaluator(s) – *single point of view evaluation*, or *local evaluation* process;
(2) the application and exploration of a *multicriteria aggregation procedure* that, taken into account some information of inter-viewpoints' nature, assembles the various criteria in an overall evaluation model – *overall evaluation* or *aggregation* process.

MACBETH (Measuring Attractiveness by a Categorical Based Evaluation Technique) is an interactive approach for quantifying value judgements about the elements of a finite set. Section 3 gives an outline of MACBETH's interactive process. In this paper, two recent real applications of multicriteria analysis in public decision-making illustrate how the MACBETH approach can facilitate either the construction of *cardinal* criteria (section 4) or the determination of the inter-criteria parameters in a given multicriteria aggregation model (section 5). Before that, section 2 briefly presents the two applications and addresses common methodological issues.

2. METHODOLOGICAL FRAME OF BOTH APPLICATIONS

The Alcântara case: The construction of a new railway link to the port of Lisbon has been postponed many times, as a result of a conflict of different viewpoints held by various public stakeholders, namely between the municipality of Lisbon (CML) and the GNFL (Lisbon Railway Node Bureau), while the Portuguese Railway Company (CP) has assumed a more flexible attitude. Over recent years, three groups of solutions, totalling 9 alternative projects, have been proposed by each of these three actors, but none has been simultaneously accepted by all the three. Recently, the development of the urbanisation plan for *Alcântara* (the zone of the city where the railway line links to the port) has presented an opportunity to try to *dissolve* the conflict, by creating a potential compromise solution that takes into account the different value systems of the stakeholders.

The Lisbon Metro case: In September 1993, the Lisbon Metro Company asked one of us to assist an internal Commission in preparing an international call for tenders for the construction of a new line for the Lisbon Metro network, and later on to facilitate the Commission's evaluation process of the tenders to decide on the contractor.

Both decision aid processes were conducted in real-time interaction with the client representatives, under a similar methodological basis and technical problematic. Once framed the problem(s), identified the actors and the type of potential actions (in the first case, projects for the new port of Lisbon railway link; in the second case, tenders for the construction of a new Lisbon Metro subway line), in both situations the *structuring* process consisted of:

1. The identification of a requisite *family* $F = \{PV_j, j = 1, \ldots, n\}$ of *fundamental points of view* for making explicit the relevant values (of the actors) for action evaluation ($n = 9$ in the *Alcântara* case and $n = 10$ in the *Lisbon Metro* case).
2. The construction, for each fundamental point of view PV_j ($j = 1, \ldots, n$), of an *impact model* for operationally describing, quantitatively or qualitatively, the degree to which the value(s) made explicit by PV_j can be satisfied. Developing such a model for PV_j led us:
   2.1. to identify or to construct a *descriptor*, that is a set X_j of *impact levels* intended to serve as a basis to describe plausible impacts of potential actions with respect to PV_j,
   2.2. to rank the impact levels of X_j in order of decreasing (local) attractiveness, that is, formally, to determine of a strict weak order $\succ_j$ modelling (in terms of PV_j) the local attractiveness of the levels of X_j for an evaluator D.
3. The definition of an *impact indicator* I_j for the identification of the *impact* $I_j(a)$ of any potential action a with respect to PV_j. Generally, and namely when uncertainty is present, an impact $I_j(a)$ is a subset (of X_j) of more than one impact level. However, in both of our cases, all the impacts of the actions could be assessed deterministically, for all fundamental points of view. Thus, for any action a and or all $j = 1, \ldots, n$, $I_j(a)$ was a single level of X_j.

We also identified for each PV_j a *neutral level of impact* x_j^0 that is an impact level considered neither attractive nor repulsive. Hereafter, a^0 will represent the reference action defined by: $I_j(a^0) = x_j^0$, for all j.

The process of *evaluating* the actions could then start. It was agreed in both cases to base the *overall evaluation* of each action a upon the construction of an additive aggregation model:

$$V(a) = \sum_{j=1}^{n} p_j . v_j(a), \quad \text{with} \quad \sum_{j=1}^{n} p_j = 1 \quad \text{and} \quad p_j > 0 \quad (j = 1, \ldots, n) \qquad (1)$$

where $v_j: A \rightarrow \Re$ is a cardinal *criterion-function*, defined on a set A of potential actions, quantifying the local attractiveness of the actions of A with respect to PV_j – our notion of *criterion-function* is close to the classical definitions of Fishburn (1978, § 2) and Roy (1985, § 9.1.2); p_j is a scaling constant for the v_j criterion, and $V(a)$ measures the *overall attractiveness* of $a \in A$. For all PV_j the zero value of the criterion-function v_j was fixed by the (unrestrictive) condition $v_j(a^0) = 0$, which of course implies $V(a^0) = 0$.

3. OVERVIEW OF MACBETH

Being A a finite set of potential actions and P the property of *attractiveness* of these actions for an evaluator D, MACBETH is an interactive approach for cardinal measurement of D's judgements of attractiveness of actions of A. The technical component of MACBETH is a chain of four linear programs, *Mc*1 to *Mc*4, first formulated in Bana e Costa and Vansnick (1994). The interested reader will find in Bana e Costa and Vansnick (forthcoming) an updated theoretical basis for our approach. The scheme in figure 1 offers a graphic overview of the cyclic interactive process of MACBETH. It starts by questioning D about the preferability he feels between any two actions a and b of A (*comparative* judgement) and, if a is judged more attractive than b (notation aPb), D is asked for a verbal *absolute* judgement about the difference of attractiveness he feels between a and b, by choosing one of six semantic categories:

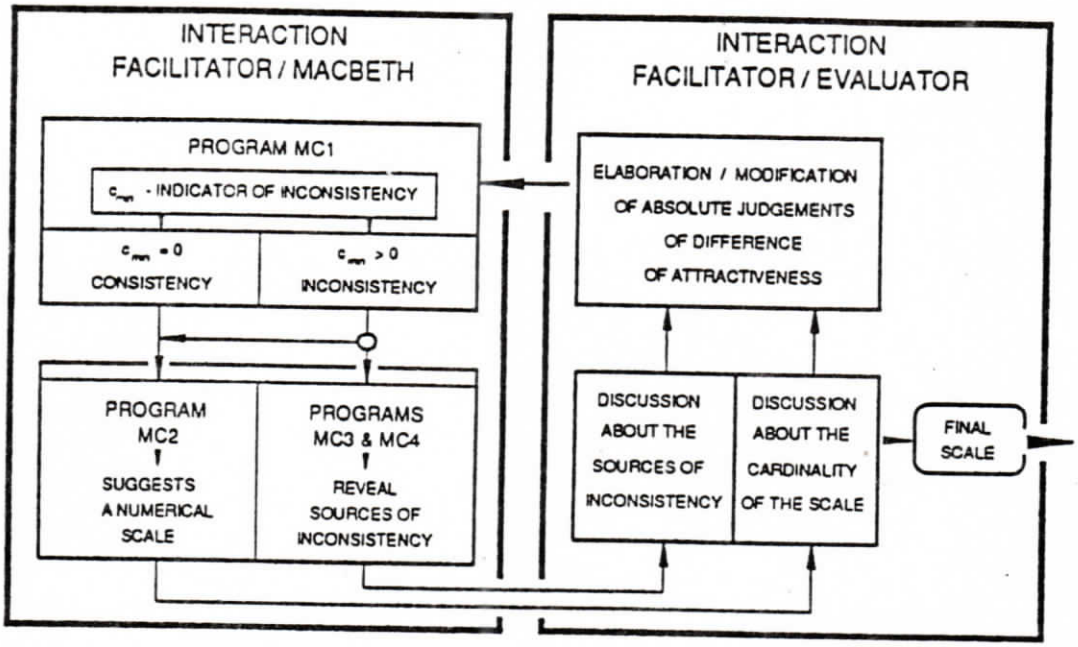

Figure 1. Interactive scheme of MACBETH

(C_1) *very weak* difference of attractiveness
(C_2) *weak* difference of attractiveness
(C_3) *moderate* difference of attractiveness
(C_4) *strong* difference of attractiveness
(C_5) *very strong* difference of attractiveness
(C_6) *extreme* difference of attractiveness.

During this questioning process, the facilitator fills in a matrix with the categorical judgements of D, analyses if it is consistent with a ranking of the actions by order of attractiveness and discusses with D until the following obvious *semantic consistency condition* is satisfied: $\forall a,b,c \in A$ with aPb and bPc and $\forall k,k' \in \{1, 2, 3, 4, 5, 6\}$, if $(a, b) \in C_k$ and $(b, c) \in C_{k'}$, one must have $(a, c) \in C_{k''}$ with $k'' \geq \max\{k,k'\}$. Applying now the chain of *programs Mc* to this judgmental information, two cases can arise according to the optimal value c_{min} of *Mc*1 (see figure 1).

Case 1: $c_{min} = 0$ (*cardinal consistency*). $c_{min} = 0$ means that it is possible to associate a real number $v(a)$ with each element a of A in such a way that the following measurement rules are satisfied (see Bana e Costa and Vansnick (forthcoming)):

Rule 1: $\forall a,b \in A$: $aPb \Leftrightarrow v(a) > v(b)$, that is, the number assigned to action a is strictly greater than the number assigned to action b if and only if, for D, a is more attractive than b.

Rule 2: $\forall k,k' \in \{1, 2, 3, 4, 5, 6\}$ with $k \neq k'$ and $\forall a,b,c,d \in A$ with $(a, b) \in C_k$ and $(c, d) \in C_{k'}$: $k > k' \Leftrightarrow v(a) - v(b) > v(c) - v(d)$, that is, when aPb and cPd, with (a, b) and (c, d) not belonging to the same category, the difference between the numbers associated to a and b is strictly greater than the difference between the numbers associated to c and d if and only if the difference of attractiveness between a and b is judged greater than the difference of attractiveness between c and d (note that in MACBETH this information is not asked directly to D, but it follows indirectly from the matrix of judgements).

In this case, *program Mc2* gives real numbers $v(a)$ ($a \in A$) that, in the above perspective, can be associated with the elements of A. These numbers can be plotted in an oriented axis (see example in figure 3) to verify with D if the relative distances between points (representing actions) correspond to relative differences of attractiveness that D feels between these actions. If so, the application $v: A \rightarrow \Re: a \rightarrow v(a)$ can be taken as an interval scale quantifying the attractiveness of the actions of A for D. If this is not the case, D is invited to modify those numbers in such a way that the above condition (relative differences) is satisfied.

Case 2: $c_{min} > 0$ (*cardinal inconsistency*). $c_{min} > 0$ means that it is not possible to represent numerically (given the measurement rules 1 and 2) the verbal judgements given by D. In this case, *program Mc2* suggests values $v(a)$ ($a \in A$) that try to "reconcile" the initial inconsistent judgements and that can be used as a basis for discussing with D the number that will be associated with each element a of A in order to satisfy the "relative differences" condition. However, as suggested in figure 1, another possibility is to invite D to re-examine his verbal judgements and to ask if he wants to modify some of them (in this case, *programs Mc1* and *Mc2* can be applied again). This interactive learning process can be facilitated by *programs Mc3* and *Mc4* for they were conceived precisely to show up certain possible sources of inconsistency and to suggest to D concrete modifications of some of his judgements with the aim of solving the inconsistency problem. Nevertheless, these suggestions must be taken simply as a basis for discussion, D being free to modify differently his initial responses. In fact, as the feedbacks in figure 1 reveal, we did not assume in the conception of MACBETH any particular purpose of convergence for the procedure just described.

Sections 3 and 4 will show, respectively, how the application of the MACBETH approach supported the construction of cardinal value functions in the *Alcântara* case, and the determination of scaling constants in the *Lisbon METRO* case.

4. THE ALCÂNTARA CASE

Following a *value-focused thinking* approach – to use the clear terminology of Keeney (1992) – a family of 9 fundamental points of view was defined, reflecting the main values favoured by each stakeholder in their projects and other values not relevant in the past, and therefore neglected in existing projects, but that the planning team considered essential to create a good solution, in the face of recent or expected modifications in other transport infrastructures within the zone.

For this presentation, we selected two points of view (PV_2 and PV_8) for which inconsistency problems arose when applying MACBETH. Table I shows the structuring information for PV_2 and PV_8. The descriptors X_2 and X_8 and their impact levels are presented in the second column of the table, together with the information about the orders $\succ_2$ and $\succ_8$. The *quantitative descriptor* X_2 chosen for PV_2 is an *indirect descriptor* (a "proxy-attribute" in the terminology of Keeney and Raiffa (1976)), whereas X_8 is a *constructed qualitative descriptor*. The levels of X_8 are presented in decreasing order of their attractiveness for the planning team, and they were defined by analysis of the new line layouts in the existing projects and other hypothetical line-schemes. The third column of the table describes the impact indicators I_2 and I_8 used to determine the impacts (see table II) of the existing projects (a_1 to a_9) in terms of PV_2 and PV_8.

The cardinal criterion-functions v_2 and v_8 were developed from the answers of the planning team (the evaluator D) to the questioning procedure of MACBETH (see tables III and IV).

Table I. Structuring information

Fundamental points of view	Impact models and neutral levels	Impact indicators
PV_2 Cut impact of the urban structure by the new line	PV_2 is operationally described by the "length in meters of the line on viaduct or in open trench, where the line crosses consolidated urban areas" $X_2 = \{x.1\text{meter} \mid 0 \le x \le 1200\}$ Neutral level: $x_2^0 = 400\text{m}$ $\succ_2$ defined by: $'x.1\text{m}' \succ_2 'y.1\text{m}'$ iff $x < y$	I_2 Lengths measured on large scale maps (the only ones available)
PV_8 Effects of the construction of the new line on the railway service to the port of Lisbon (note that PV_8 combines two elementary means points of view that are *judgementally dependent* – see von Winterfeldt and Edwards (1986, p. 42)	PV_8 is operationally described by a five level scale defined by different possibilities of splitting up the construction of the new line into track-sections, while keeping (or not keeping) in operation the old railway line $X_8 = \{++, +, o, -, --\}$ Neutral level: $x_8^0 = o$ $\succ_8$ defined by: $++ \succ_8 + \succ_8 o \succ_8 - \succ_8 --$ where: ++ It is possible to split up the construction of the new line in all its track-sections, while keeping in operation the old line + It is possible to split up the construction in only two track-sections, while keeping in operation the old line o It is impossible to split up the construction, while keeping in operation the old line − It is possible to split up the construction in only two track-sections, but it is impossible to keep in operation the old line −− It is impossible either to split up the construction, or to keep in operation the old line	I_8 Analysis of the projects by the civil engineers of the planning-team

Table II. Impacts of the alternative projects with respect to PV_2 and PV_8

Impacts	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
$I_2(.)$ (meters)	1200	1020	400	260	300	580	700	1120	0
$I_8(.)$	o	—	o	o	o	o	o	−	o

Table III. Judgements of difference of attractiveness by PV_2

→	a_4	a_5	a_3	a_6	a_7	a_2	a_1
a_9 (0 m)	4	5	5	5	6	6	6
a_4 (260 m)		1	2	3	4	5	6
a_5 (300 m)			2	2	4	4	5
a_3 (400 m)				2	2	4	4
a_6 (580 m)					2	3	4
a_7 (700 m)						2	2
a_2 (1020 m)							1

Table IV. Judgements by PV_8

→	+	o	−	−−
++	3	3	5	6
+		4	4	6
o			4	5
−				4

Note in table IV that D assigned each pair of impact levels, not of projects, to one category C_k ($k \in \{1, 2, 3, 4, 5, 6\}$) of difference of (local) attractiveness by PV_8, because the set of potential projects is *unstable*, in the sense that, a future consensus layout may have an impact with respect to PV_8, for instance +, different from the impacts $I_8(.)$ of the existing projects shown in table II. For PV_2, the judgements could be given directly in the set A of the existing projects, once X_2 is continuous and no new project can have an impact out of the scale range defined by A.

When value reasoning about the impacts of the projects in terms of effects on the urban structure (PV_2) D felt insecure in judging the difference of attractiveness between two projects for which the difference in lengths of line is lower than $\delta_2 = 10\%$ of the longer of the two; Roy (1985) called δ_2 a *dispersion threshold*. One reason for this attitude of D is that X_2 is an indirect descriptor, which only describes in a rough manner the actual impact caused by the railway line on the urban structure. Moreover, there is a significant degree of imprecision in the lengths of the new railway lines in the projects, for, as said in table I, they were measured on large scale maps. Once $I_2(a_1) - I_2(a_8) = 80\text{m} < 0.10 \times 1200\text{m}$ and $I_2(a_8) - I_2(a_2) = 80\text{m} < 0.10 \times 1120\text{m}$ (see table II), a_8 was accordingly kept out of the initial questioning process (see table III).

Starting with the initial judgements by PV_2, let us explain how the MACBETH interactive learning process moved forward. First, we verified that the set of judgements in table III is *semantically consistent* (indeed, the categories never decrease from left to right in the lines and never increase from top to bottom in the columns). Nevertheless, the optimal solution of program *Mc*1 is $c_{min} = 1$ ($\neq 0$) which means that set of judgements is not cardinal consistent. Accordingly, we (the facilitator) asked D if he was sure, or not, of all of his initial judgements. D said he had problems when judging the difference of attractiveness between a_9 and a_5, hesitating between *very strong* or *strong*. Remember that $(a_9, a_5) \in C_5$ means that D judged *very strong* the difference of attractiveness between no cut impact of the urban structure by the new line ($I_2(a_9) = 0$ m) and a cut impact of 300 meters length ($I_2(a_5) = 300$ m, see table II). The hesitation was due to the fact that D judged only *strong* an increase from 0 m to 260 ($(a_9, a_4) \in C_4$). After some discussion, D decided to revise his initial judgement to $(a_9, a_5) \in C_4$. It is interesting to note that this was precisely one of the suggestions of modification proposed by *Mc*4.

At this point in evolution of the process, we turned again to MACBETH. The revised set of judgements is now consistent ($c_{min} = 0$). Accordingly, the discussion went on to the validation with D of the cardinality of the numerical scale v given by *Mc*2. That scale v has been linearly re-scaled to set the cardinal criterion-function $v_2 : A \rightarrow \Re$ so that $v_2(a^0) = 0$ and v_2(local best alternative) = 100. Observe the convex shape of the curve and note that, if $I_2(a_6) = 550$m instead of the actual 580m (a difference of length lower than the dispersion threshold δ_2) the function v_2 would be most probably also convex between a_3 and a_7. Finally, the scale has been accepted, and we defined the value $v_2(a_8)$ by linear interpolation between $v_2(a_2)$ and $v_2(a_1)$.

Let us now resume the interactive path towards the construction of a cardinal criterion for PV_8. By simple observation of table IV, one may verify that the semantic consistency condition is violated by the sequence of judgements $(++, +) \in C_3$, $(+, o) \in C_4$ and $(++, o) \in C_3$. When confronted with this inconsistency problem, D made the comment that he felt less confortable in judging attractiveness regarding the fundamental point of view PV_8 than for the other ones. This is not at all surprising, for PV_8 is a complex point of view combining two dependent elementary ones (see table I). After some thought, D concluded that $(++, o) \in C_3$ was quite biased and decided to modify it to *strong*. Note that this solves the semantic inconsistency problem. As no other modification has been introduced by D, we moved forward to the analysis of the cardinal consistency of the revised set of judgements, with the aid of *Mc*1. As its optimal solution is $c_{min} = 0$, the discussion went on to the validation of the numerical scale v suggested by *Mc*2 (see figure 3).

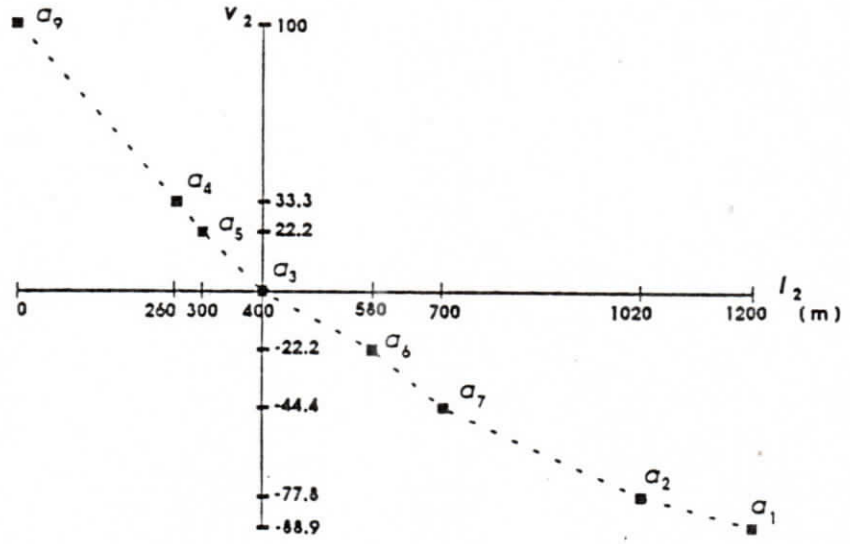

Figure 2. Criterion-function by PV_2

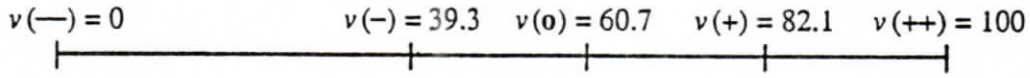

Figure 3. MACBETH's scale with respect to PV_8

Finally, the values of the scale v being accepted by D, we determined the corresponding cardinal criterion-function $v_8 : A \to \Re$ by: $v_8 = g(v(I_8))$, where g is a positive linear transformation such that $v_8(a^0) = g(v(o)) = 0$ and v_8(local best alternative) = $g(v(++)) = 100$:

$$\begin{cases} v_8(a_1) = v_8(a_3) = v_8(a_4) = v_8(a_5) = v_8(a_6) = v_8(a_7) = v_8(a_9) = 0 \\ v_8(a_8) = -55 \\ v_8(a_2) = -155. \end{cases}$$

It was interesting to note that, accordingly to criterion v_8, the projects a_2 and a_8 are significantly repulsive in terms of effects of the construction of the new line on the railway service to the port of Lisbon (PV_8). So, in creating a new layout, the characteristics of these two ones should be avoid.

5. THE LISBON METRO CASE

In this case, the MACBETH approach has been used to facilitate the determination of the scaling constants ("weights", hereafter) in an additive aggregation model 1, which is the aggregation procedure used in the Company to evaluate tenders, and so, the model was imposed *a priori*. The definition of the weights was a quite important issue because, by imposition of both Portuguese and European laws, the so-called "appreciation criteria" (our fundamental points of view) must be enunciated in the call for tenders by "order of importance". Moreover, the Company, invoking transparency in decision-making, traditionally divulges the numerical values of the "weights" already in the calls for tenders, although the law does not compel to go so far.

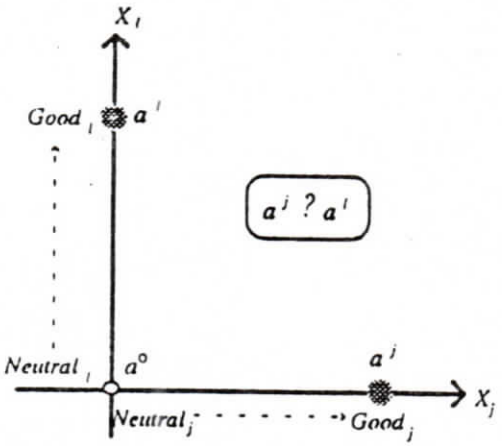

Figure 4 MACBETH's swings

Therefore, a particular problem to be addressed was how to determine the scaling constants in a theoretically sound way without knowing the ranges of the impact scales, since, when preparing a call for tenders, obviously one does not know either how they will be or their proposals! The evaluation Commission (D, hereafter) is formed of three engineers and two economists who top the technical hierarchy of the Company, and who did not escape that so common mistake of determining (additive) weights without reference to the impact scales; Keeney (1992, § 5.4) calls it "the most common critical mistake". So, the first task has been to explain to them that the discrimination power of those scales must be taken into account in a proper weighting procedure, provided one wants to evaluate the tenders by an additive model.

The weighting procedure adopted consists in anchoring on two impact levels not dependent on a particular set of actions, and to determine the weights indirectly by applying MACBETH to holistic semantic judgements of difference of overall attractiveness between fictitious actions defined by the anchor levels. For each j from 1 to n, let $x_j^+ \in X_j$ be a *good* level of impact in terms of PV_j and $x_j^0 \in X_j$ a *neutral* level, as defined in section 2. Recall that a^0 is such that $I_j(a^0) = x_j^0$ for all j = 1, ..., n, and let also a^j (j = 1, ..., n) be the fictitious action defined by: $I_j(a^j) = x_j^+$ and $I_l(a^j) = x_l^0$ for all $l \neq j$. Finally, define a reference set $A^F = \{a^0, a^1, ..., a^j, ..., a^n\}$ of fictitious actions. Formally, MACBETH's weighting questioning procedure consisted in asking D: first, to compare all two actions a^j and a^l in A^F in terms of *overall* attractiveness; second, to judge semantically the difference of overall attractiveness between any two reference actions such that the first one is more attractive than the second one. (The same could be done for reference actions defined in terms of best or worst levels, as it is more usual.). In practice, in each question (see figure 4) D is simply asked to confront the difference of attractiveness he feels between a swing of impact from x_j^0 to x_j^+ in PV_j and a swing of impact from x_l^0 to x_l^+ in PV_l (with all the other impacts being identical). This procedure does not automatically imply consistency of the answers of D, contrary to both *trade-off* procedure of Keeney and Raiffa (1976) and *swing* weighting of von Winterfeldt and Edwards (1986). The judgements expressed by D are shown in table V (n = 10).

Note that the rows and the columns of a^2 and a^1 on the one hand, and of a^8 and a^{10} on the other hand, are very similar. During the questioning procedure, D hesitated in considering the actions of these two pairs as indifferent, but he agreed to wait for the suggestions of *MACBETH*. Observing in table V that semantic consistency is verified, we applied to it the programs *Mc*.

Table V. Matrix of judgements, with no *ex-aequo*

→	a^2	a^1	a^6	$a^{4=5=9}$	a^8	a^{10}	$a^{3=7}$	a^0
a^2		1	4	4	6	6	6	6
a^1			4	4	5	5	6	6
a^6				4	5	5	5	5
$a^{4=5=9}$					3	3	4	4
a^8						1	2	4
a^{10}							2	3
$a^{3=7}$								2
a^0								

The output of *Mc*1 is $c_{min} = 1$ (> 0), revealing problems of cardinal inconsistency. Nevertheless, in this case, other than analysing the sources of inconsistency, *D* has preferred to discuss directly the scale proposed by *Mc*2, which is: $v(a^2) = v(a^1) = 100$, $v(a^6) = 75$, $v(a^4) = 50$ (= $v(a^5) = v(a^9)$), $v(a^8) = v(a^{10}) = 25$, $v(a^3) = 12.5$ (= $v(a^7)$) and $v(a^0) = 0$. Finally, *D* decided for indifference between a^2 and a^1, and between a^8 and a^{10}. Moreover, the scale given by *Mc*2 has been considered to satisfy the test of "relative differences" (see section 3).

The final respective values of the scaling constants are: $p_2 = p_1 = 0.20$, $p_6 = 0.15$, $p_4 = p_5 = p_9 = 0.10$, $p_8 = p_{10} = 0.05$ and $p_3 = p_7 = 0.025$. These values were directly determined from scale *v*, for applying model 1 to each reference action a^j results:

$$p_j = \frac{v(a^j)}{\sum_{j=1}^{10} v(a^j)}.$$

6. EXTENSIONS

In the practice of decision-aid, a facilitator should never consider the process finished without discussing the sensitivity/robustness of the recommendations emerging from the application of a certain approach, especially when single numerical values were assigned to the parameters included in the model (such as scores of relative attractiveness or weights of criteria). Our on-going research on the MACBETH approach is precisely oriented towards the above type of concerns. For instance, in a framework of classical sensitivity analysis, it is easy to obtain an interval of variation for each numerical value individually, respecting a consistent set of semantic judgements.

REFERENCES

Bana e Costa, C.A. and Vansnick, J.C. (1994), "MACBETH – An interactive path towards the construction of cardinal value functions", *International Transactions in Operations Research*, 1, 4, 489–500.

Bana e Costa, C.A. and Vansnick, J.C. (forthcoming), "A theoretical framework for Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH)", J. Clímaco (ed.), *Proceedings of the XIth International Conference on MCDM*, Coimbra, Portugal, August 1994.

Fishburn, P.C. (1978), "A survey of multiattribute/multicriterion evaluation theories", in S. Zionts (ed.), *Multiple Criteria Problem Solving*, Springer-Verlag, 181–224.

Keeney, R.L. (1992), *Value-Focused Thinking: A Path to Creative Decisionmaking*, Harvard University Press.

Keeney, R.L. and Raiffa, H. (1976), *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*, John Wiley & Sons.

Roy, B. (1985), *Méthodologie Multicritère d'Aide à la Décision*, Economica.

Von Winterfeldt, D. and Edwards, W. (1986), *Decision Analysis and Behavioral Research*, Cambridge University Press.