6.252, Spring 2003, Prof. D. P. Bertsekas
Midterm In-Class Exam, Closed-Book, One Sheet of Notes Allowed

Problem 1: (30 points)

(a) Consider the method $x^{k+1} = x^k + \alpha^k d^k$ for unconstrained minimization of a continuously differentiable function $f : \Re^n \mapsto \Re$. State which of the following statements are true and which are false. You don't have to justify your answers:

1. If $d^k = -\nabla f(x^k)$ and α^k is such that $f(x^{k+1}) < f(x^k)$ whenever $\nabla f(x^k) \neq 0$, every limit point of the generated sequence $\{x^k\}$ is stationary.
2. If $d^k = -\nabla f(x^k)$, α^k is chosen by the Armijo rule, and the function f has the form $f(x_1, x_2) = (x_1)^2 + (x_2)^2 + x_1$ the generated sequence $\{x^k\}$ converges to a global minimum of f.

(b) Consider the minimization of $f(x) = \|x\|^2$ subject to $x \in X$ where $X = \{x \mid x_1 + \cdots + x_n = 1\}$. State which of the following statements are true and which are false. You don't have to justify your answers:

1. The conditional gradient method with some suitable stepsize rule can be used to obtain a global minimum.
2. The gradient projection method with the line minimization rule can be used to obtain a global minimum, and converges in a single iteration.
3. The constrained version of Newton's projection method with stepsize equal to 1 can be used to obtain a global minimum, and converges in a single iteration.

Problem 2: (35 points)

Consider the 2-dimensional function $f(x, y) = (y - x^2)^2 - x^2$.

(a) Show that f has only one stationary point, which is neither a local maximum nor a local minimum.

(b) Consider the minimization of f in part (d) subject to no constraint on x and the constraint $-1 \leq y \leq 1$ on y. Show that there exists at least one global minimum and find all global minima.

Problem 3: (35 points)

Among all parallelepipeds with given sum of lengths of edges, find one that has maximal volume. Are the 2nd order sufficiency conditions satisfied at the optimum?